NOAA Technical Report NOS NGS 62

Blueprint for 2022, Part 1: Geometric Coordinates

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Executive Summary

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In 2022, the entire National Spatial Reference System (NSRS) will be modernized. This document addresses the *geometric* aspects of the NSRS. Geometrically, the NSRS currently contains three reference frames (historically "horizontal datums"), known as NAD 83(2011), NAD 83(PA11) and NAD 83(MA11) which are used to define the geodetic latitudes, geodetic longitudes and ellipsoid heights of all points in the USA. These three frames will be replaced with four new reference frames, called:

- *North American Terrestrial Reference Frame of 2022 (NATRF2022)*
- *Pacific Terrestrial Reference Frame of 2022 (PTRF2022)*
- *Caribbean Terrestrial Reference Frame of 2022 (CTRF2022)*
- *Mariana Terrestrial Reference Frame of 2022 (MTRF2022)*

The time-dependent Cartesian coordinate of any point on Earth in any of these frames [*x,y,z*] will be defined as: (a) identical to (at epoch t_0) and (b) relative to (at epoch $t=t_0+\Delta t$) the time-dependent Cartesian coordinates in the latest pre-2022 global reference frame [*X,Y,Z*] from the International GNSS Service (IGS). The relative relationship over time will rely on an NGS-determined plate rotation model for each tectonic plate associated with each frame. This relationship will resemble a traditional 14 parameter transformation, but only three (time-dependent rotations about the three IGS axes) will be non-zero.

Such time-dependent coordinates will exhibit spatial stability in areas of the continent where motion of the tectonic plate is fully characterized by plate rotation. All remaining velocities (including horizontal motions induced directly or indirectly by adjoining tectonic plates, horizontal motions induced by Global Isostatic Adjustment, other horizontal motions and all vertical motions in their entirety) will be captured by an Intra-Frame Velocity Model (IFVM). Such a model will allow users to compare time-dependent coordinates in any of the four terrestrial reference frames, across years.

The use of the IFVM will allow NGS to provide, as a primary service, time-dependent coordinates at the highest levels of accuracy, while subsequently providing a secondary service of comparing those time-dependent coordinates across time at lower levels of accuracy. However, this document does not yet define the exact 'fabric' or delivery of the IFVM, only its definitive part in the NSRS and its expected initial role.

The ellipsoid used to relate Cartesian coordinates to geodetic coordinates will be GRS-80.

April 21, 2017

<u>Blueprint for 2022: Part 1, Geometric Coordinates</u>

1 Purpose

The intent of this document is to provide to the public the current status of plans by the National Geodetic Survey (NGS) to modernize the National Spatial Reference System (NSRS) in 2022. This particular document covers the Geometric component; that is, the definition and determination of latitude, longitude and ellipsoid heights.

This document does not attempt to be comprehensive, but it is being released with the express intent of stating what is currently known, while leaving some items "to be determined" (TBD). As feedback is collected about this document, further refinements to this blueprint will be made. It is expected that updated releases of the blueprint will occur both before 2022 and shortly thereafter as more details become codified.

Therefore, a word of caution is appropriate: Many portions of this document are purposefully vague. NGS requests and welcomes feedback from the user community, particularly on those aspects which still have vague, TBD information.

2 Introduction

The mission of the National Geodetic Survey (NGS) is to define, maintain and provide access to the National Spatial Reference System (NSRS), to meet our nation's economic, social, and environmental needs. The NSRS is defined by the Office of Management and Budget's (OMB) circular A-16 (Coordination of Geographic Information and Related Spatial Data Activities) as "the fundamental geodetic control for the United States" and is required to be used by all federal government agencies creating geographic information within the United States.

In order to keep up with changing technology and improved accuracy, NGS has planned for a modernization of the NSRS by 2022. In order that this modernization maintains the usefulness of the NSRS, the *function* of geodetic control should be clearly articulated first.

3 "Geodetic Control"

According to OMB A-16, "geodetic control provides a common reference system for establishing coordinates for all geographic data." That is, geodetic control is some system which allows users to determine the latitude, longitude, height, gravity or other coordinate at points in their geographic dataset in such a way that these coordinates are *consistent with* similarly derived coordinates prepared by other users using other datasets, but using the same geodetic control. Therefore, geodetic control must be more accurate than any map or other data set built upon it. There is no unanimous definition of threshold values that define "geodetic accuracy" or "mapping accuracy"; this is especially true considering (for example) that the state-of-the-art positioning accuracy was about 1 meter just a few decades ago, but now it is in the centimeter and even millimeter range. Therefore, while terms like "geodetic accuracy" or "mapping accuracy" (or "geodetic or mapping 'quality'")) may be used in this document, they should be taken relative to one another, rather than in an absolute sense. Geodetic accuracy should be considered state-of-the art positioning accuracy, while mapping accuracy is anything less accurate than that, but still capable of providing useful information in many map applications or other geospatial products, such as boundary and engineering surveys.

Unfortunately missing from this functional statement is the reality that geodetic control points (and their respective coordinates) can, and do, move over time. A significant portion of this blueprint will be dedicated to addressing why this is true and what can be done about it.

In order to fulfill its function, classical geodetic control was usually a network of metal disks or rods affixed to the surface of the Earth with some associated coordinates such as latitude, longitude, height or gravity, and where such coordinates are mutually consistent within the network. Such points served as "starting points" for the *users* of geodetic control to begin their own surveys and thus create their own maps or other geographic datasets. By requiring all federal creators of geographic data to use the same geodetic control network (the NSRS), all geographic data in the USA created at the federal level should therefore be mutually consistent.

As technology has progressed, our ability to establish accurate positions has outpaced the accuracy of our underlying geodetic control. Coordinates do change over time due to a variety of factors operating over different spatial and temporal scales. In general, these scales were either spatially small or temporally very long, and were of a magnitude smaller than the accuracy of the surveys which created the coordinates. For example, on a typical engineering timescale, coordinate drift is typically less than the aforementioned 1 meter state-of-the-art absolute accuracy of the mid-late 20th century. Therefore, it was possible for geodetic control to function for decades with the assumption of "fixed" coordinates, only occasionally getting updated in certain locations when movement, exceeding the accuracy of existing surveys, was finally detected. That all changed in the 1980s with the advent of the Global Positioning System (GPS) and other space geodetic techniques. These new positioning technologies, with their ability to measure baselines thousands of kilometers in length to a few centimeters of accuracy, began to detect (and thus validate the theory of) tectonic plate drift. A variety of approaches to providing geodetic control have been attempted since then, including:

1) Global, plate-independent reference frames, such as the International Terrestrial Reference Frame (ITRF), which embraces time dependency as part of geodetic control. [1]
2) "Plate Fixed Frames", such as NAD 83, which attempt to "affix" a coordinate frame (at least in latitude and longitude) to one tectonic plate so as to maintain unchanged coordinates on that plate. This approach comes with its own assumptions, such as the rigidity of the tectonic plate.

Neither of these approaches presents a perfect solution to reconcile the considerations and capabilities of the geodetic control provider with the practical expectations of the geodetic control user community. For instance, many surveyors still have equipment, software and other tools which presume that geodetic control remains "fixed" (constant) in time. This simplifies project planning and computations

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[1] More recent global initiatives, such as the United Nations Global Geospatial Information Management (UN-GGIM) have raised the ITRF from an independent scientific project to a UN supported global initiative.

significantly, and also aligns with the majority of geodetic control services historically provided by NGS within the NSRS. But it ignores the true nature of the Earth by oversimplifying geospatial data collected at different points in time and limiting the ability to combine datasets that cover very large geographic areas. Although this situation is changing, not all users of geodetic control can readily adapt to a system where coordinates change in time. As such, some compromise is necessary for practical purposes when modernizing the NSRS.

One type of compromise between the users of geodetic control and the providers of such control is through the definition of a “plate fixed” reference system, rather than a global (plate-independent) reference system. Such a compromise breaks down in areas where a tectonic plate is not completely rigid, where it is not moving in a uniform or predictable manner, or where complex intraplate motion is present. However, as a first approximation, a “plate fixed” system is an incredibly useful compromise because it can cover large portions of a tectonic plate, provide accuracies over time which are acceptable to many geographic data providers, and it can be easily implemented. As will be discussed later, once the plate rotation is removed, a significant portion of the country will experience small (to the point of being negligible) time dependent motions; and even those portions of the country which experience large motions besides the rotation of one particular plate will have those motions modeled separately (see Section 8). As such, the use of “plate fixed” reference frames was chosen by NGS as part of the NSRS modernization.

No matter its nature (passive or active), the purpose of geodetic control is to provide starting points by which geospatial users. may define positions with the consistency and reliability of the National Spatial Reference System. Such starting points should have known coordinates at the epoch when the geospatial professionals are using that control. If those coordinates have changed over time, then it would be convenient if some component of the geodetic control should allow for comparison of previously determined geospatial coordinates at different epochs.

4 “Plate Fixed” and Euler Poles

It was only a century ago that “continental drift” was first proposed (Wegener, 1915), but it wasn’t until the 1950s that enough evidence of “plate tectonics” began to accumulate that in the 1970s it became an accepted, proven theory. Today, it is recognized that the motion of many plates is not best characterized by “drifting,”, but could more accurately be described as “rotating”. The horizontal motion of many points on a tectonic plate (relative to a global ideal reference frame like the ITRF) can be modeled as a rotation about a geocentric axis passing through a fixed point on Earth’s surface. Although such models must make certain assumptions (such as the rigidity of the plate), the dominant motion of the majority of points on most tectonic plates is the rotation about a fixed point. That point is known as an “Euler Pole”. See Figure 1. The determination of a plate’s Euler Pole location and the angular velocity with which a plate rotates can be empirically determined through the analysis of years (even decades) of GNSS observations distributed around the plate. With longer time series, wider geographic distribution and the accurate modeling of non-Eulerian motions, the knowledge of the plate’s rotation improves.

Under the presumption that plate-wide small (relative) magnitude horizontal motions like GIA are properly modeled and removed from the otherwise rigid parts of a tectonic plate, plates can be assumed to have effectively non-deforming (rigid) portions. These portions of the plate are generally in the interior, and if this part of the plate is truly rigid, points on these portions of the plate do not move relative to one another. This discussion will restrict itself solely to that part of a tectonic plate which exhibits rigidity.

If one examines the global motion of the rigid part of a tectonic plate, it is often the case that the motion looks like the plate is being rotated about some geocentric axis passing through a fixed point on the Earth. The Euler Pole is usually not on the plate itself, but the rotation about that pole should be constant (often expressed in angular velocity units such as degrees of rotation per million years or milli-arc-radians per year). This means that, viewed from a purely horizontal motion standpoint, points nearer the Euler pole seem to be moving slower (in linear velocities, like centimeters per year) and points further from the Euler pole appear to be moving faster (again, in linear velocities like centimeters per year), but in truth, they are all moving at the same *angular* velocity.

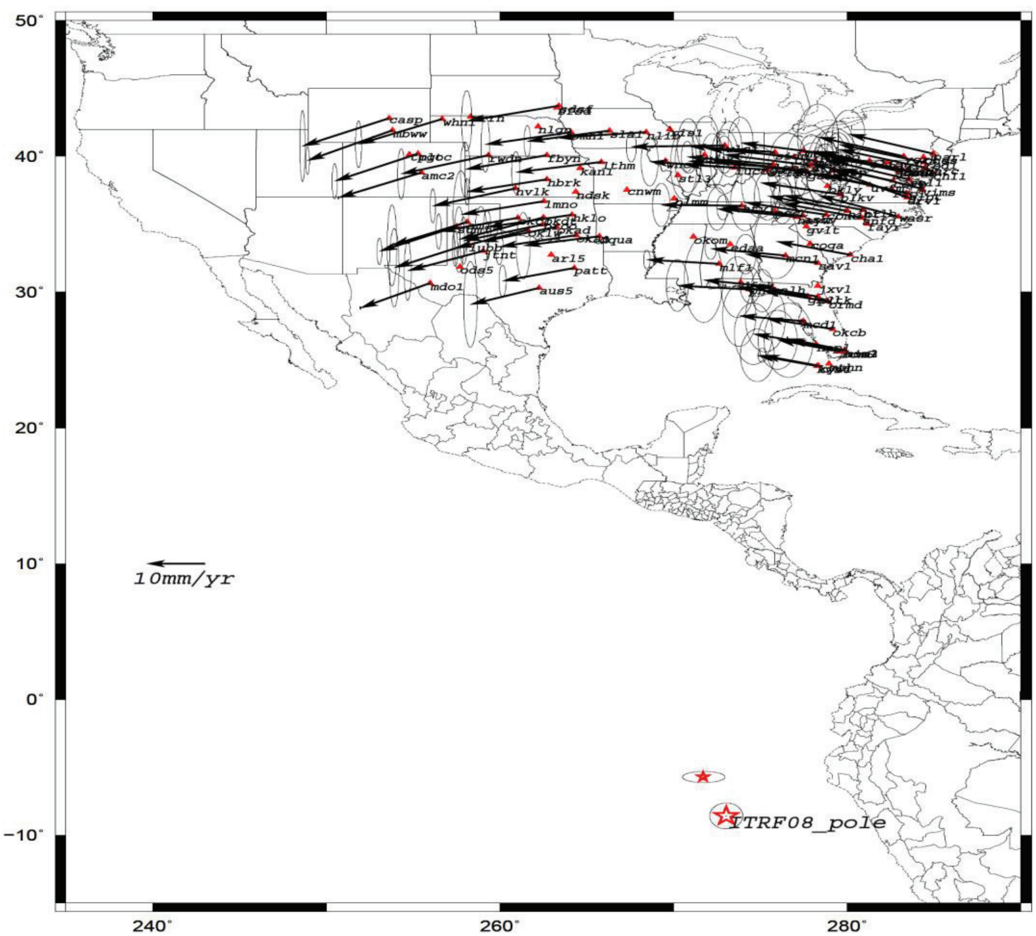


Figure 1: Vectors of horizontal velocity at 114 Continuously Operating Reference Stations (CORS) used in the "repro1" solution at NGS, as well as its associated Euler Pole solution, for the North American Plate. Also shown, for comparison, is the ITRF08 Euler Pole solution. Error ellipses are also shown to represent the uncertainty in both the magnitude and azimuth of the velocity vector.

However, no tectonic plate is perfectly rigid. When the motions seen in Figure 1 are removed from the measured horizontal velocities at *any* CORS station in North America, non-Eulerian motions are detected. These non-Eulerian velocities are shown for the Eastern and Western parts of CONUS in Figures 2 and 3.

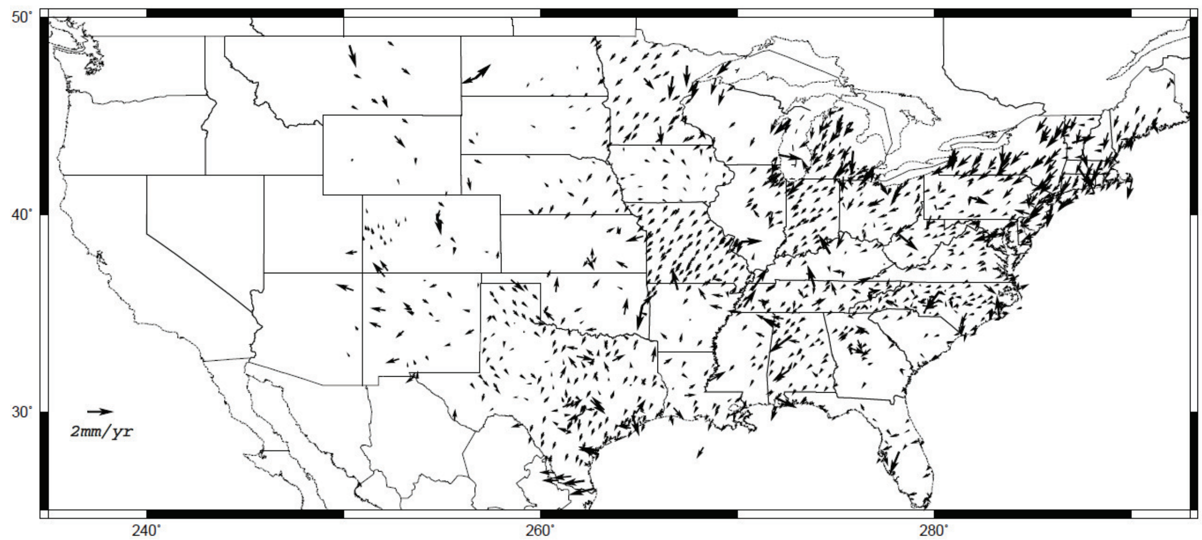


Figure 2: Repro1 horizontal non-Eulerian velocities (observed – Euler-derived) to the east of longitude 110W. Their magnitude is smaller than 2 mm/year. It is expected that those stations which were used to derive the Euler Pole will behave well (have small non-Eulerian velocities) while other stations may have larger non-Eulerian velocities.

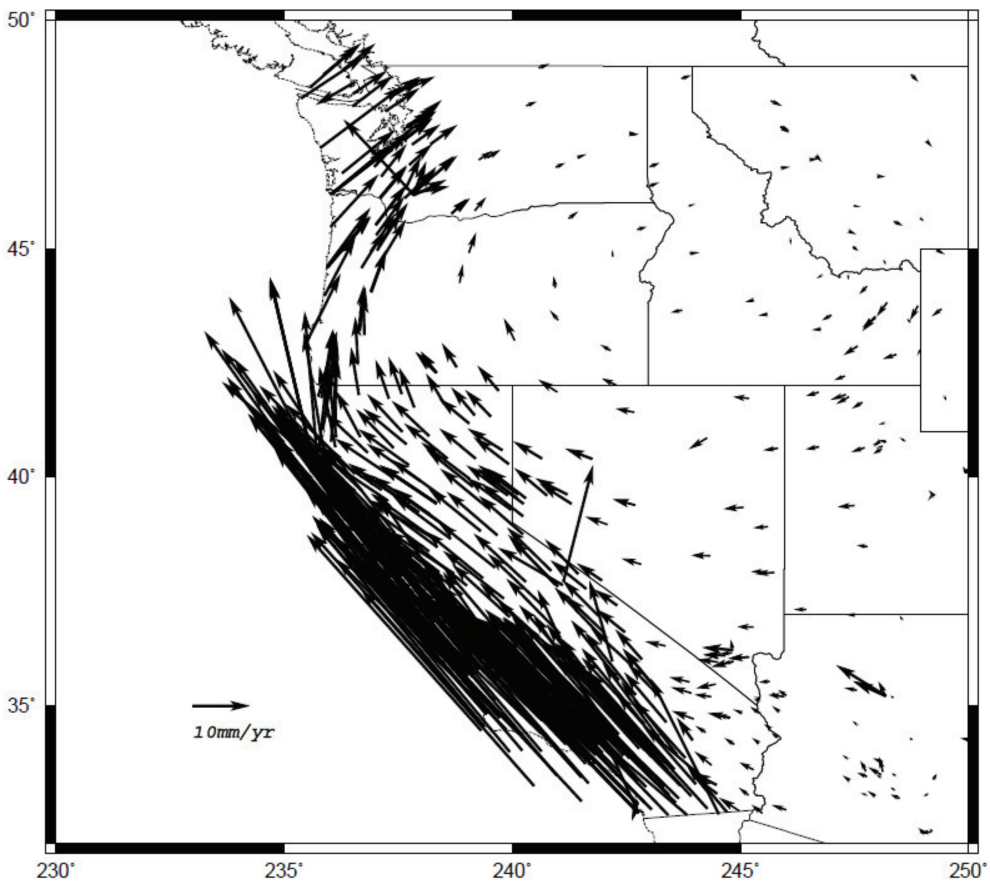


Figure 3: Horizontal non-Eulerian velocities in the west of longitude 110W (the result of removal of the rotation of the North American plate). The large vectors in Western California are points on or near the Pacific Plate while the larger vectors in Western Oregon and Washington show areas of deformation near plate boundaries, all of which therefore exhibit velocities which cannot be adequately captured just from the North American plate rotation.

Figure 2 appears to have mostly random scatter, but a close look at some areas, such as the Northeast, shows that some of this non-rigid motion is systematic as well. Based on the non-zero size of the resultant non-Eulerian vectors (random or systematic), real-world "plate fixed" coordinates cannot be simply defined as being affixed to a rigid plate. Therefore, an interpretation of "plate fixed" coordinates may fall into one of two categories, one that treats the plate as entirely static and one that allows coordinates to follow some fixed characteristic motion that is specific to the plate

In the first interpretation, a "plate fixed" coordinate system *could* mean that coordinates on a tectonic plate never change over time. This interpretation carries some simplicity, as one need only fix the coordinates of all active and passive control in a frame at some reference epoch. This then means that, by definition, all vectors between any two points are also permanently fixed in size and direction. This immediately introduces some difficulties within the real world, including:

a) *All* points actually have *some* non-Eulerian motions (see Figures 2 and 3), so that actual measurements of vectors over the years between points will not agree with the fixed vectors nor with any other measurements at any other survey epochs. However, these discrepancies may be caused by actual motion or by survey error. Therefore, this definition requires either the surveyor or the maintainer of the frame provide geodetic quality models of time-varying 3-D motions at all existing geodetic control points, as well as future geodetic control points, so that true motion and survey measurement error may be distinguished from one another. This ultimately means a 3-D motion model for the entire continental crust must be available, as one cannot ever know where future geodetic control might be installed, and a newly installed mark, tied to pre-existing control, will need to be relatable through time to the location of that pre-existing control at a previous epoch.
b) Plate boundaries are not always obvious, so knowing whether a point should or should not be fixed to a particular plate may be difficult.

In the second interpretation, one might assume that a tectonic plate is "rigid" and that points upon that plate only move due to rotation of that rigid plate. This also presents additional difficulties for application in the real world, including:

a) Points which are on any non-rigid part of the plate (usually near compression zones near plate boundaries; see Figure 3) will have significant velocities which are not captured by a plate rotation
b) Small, but noticeable, horizontal motion may occur in association with significantly large vertical signals (such as Glacial Isostatic Adjustment).
c) *All* points actually have *some* non-Eularian motions (see Figure 2), so removing just plate rotation will still yield points that move through time.
d) *The rigidity assumption also assumes that no vertical movement is happening.*

A word of caution before proceeding: the assumption of a tectonic plate being "rigid" is a reasonable first approximation in the interior of many plates, but cannot be taken as absolute. The most obvious deviation from this comes near the boundaries of two plates where non-rotational motion comes in the form of compression or other deformation. But there is one other signal which can span large portions of an otherwise "rigid" plate, effectively nullifying true rigidity, and that is the horizontal signal associated with glacial isostatic adjustment (GIA). To envision this, think of the North American Plate as a flat bedsheet. If one pinches the sheet at Hudson Bay and begins lifting vertically, then all points on the sheet begin to slide horizontally (radially) toward Hudson Bay. One such model of this motion is seen in Figure 4.

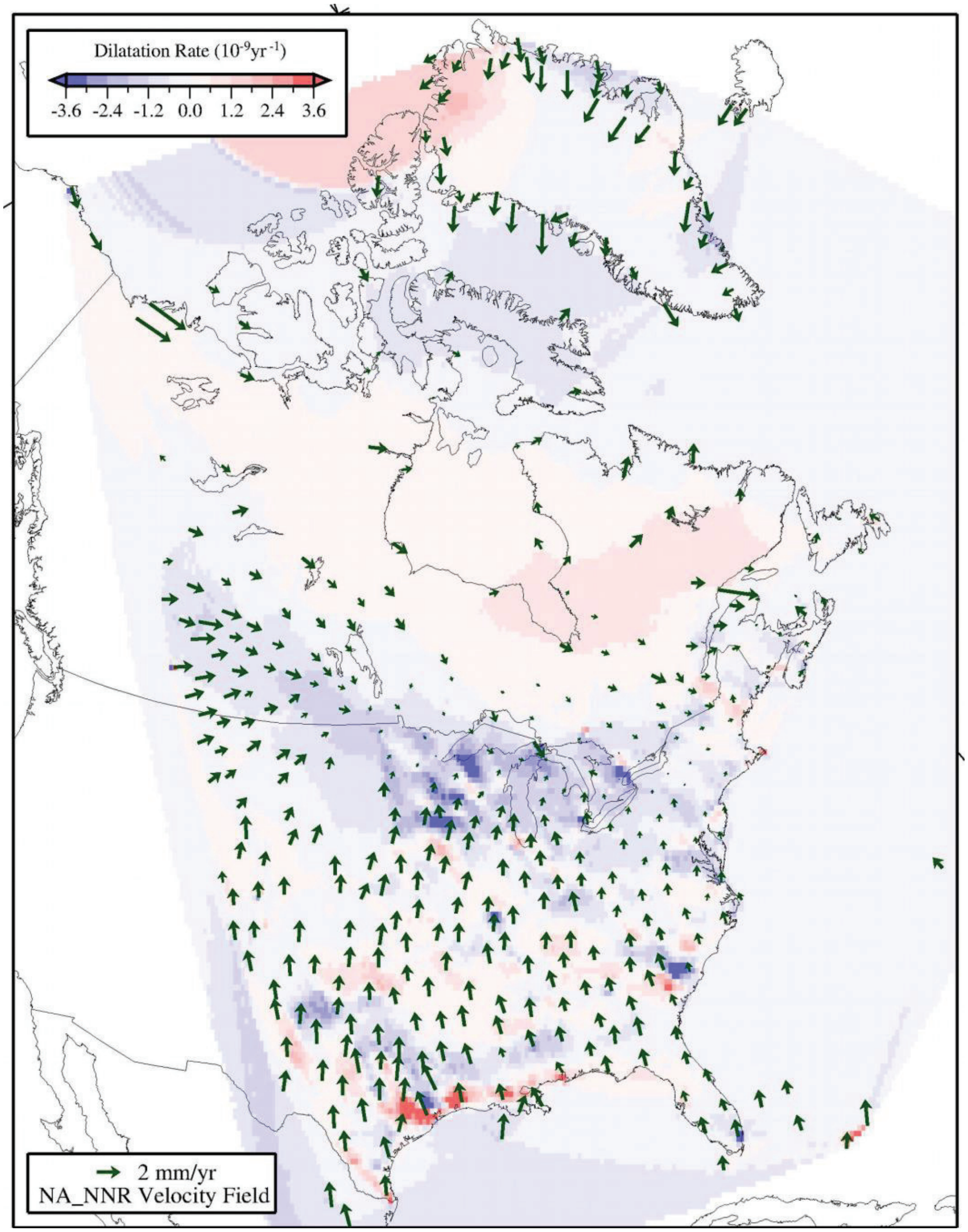


Figure 4: GIA-specific horizontal non-Eulerian velocities (Euler Pole Rotation Removed) using the MELD model (Blewitt, et al, 2016)

Despite the fact that the overwhelming majority of horizontal motion on the North American Plate may be described by rotation about a fixed Euler Pole, the measurable amount of horizontal motion that is an artifact of GIA centered around a few nodes on the plate (the largest being at Hudson Bay) cannot be ignored. However, it does not lend itself well to the simple mathematical description that comes with an Euler Pole rotation. As such, it is worth remembering that to speak of a "rotation of a rigid plate" one must make assumptions about how potential plate-wide non-rotational motions will be handled.

Therefore, to best characterize the motion of geodetic control within the NSRS, we utilize a hybridized approach of the "fixed coordinates forever" and "the entire plate is rigid" interpretations described above. The modernized NSRS will contain four "plate fixed" terrestrial reference frames, one for each of four different plates (North American, Caribbean, Pacific and Mariana), for which the term "plate fixed" will mean that the Euler Pole rotation of the plate, uncorrupted by any other systematic or random horizontal motions, will be calculated and used to define the **mathematical** relationship of latitude and longitude between an ideal global reference frame (such as the ITRF) and each of the four terrestrial reference frames of the NSRS. To put it another way, for each of the four plates, a rigid *frame* (of latitude and longitude) will be created which will rotate about the best determined Euler Pole for that plate at the best determined angular velocity for that plate. NGS will work to re-establish an International Association of Geodesy (IAG) working group specifically to address the determination of the four Euler Poles needed. While the North American and Pacific plates already have well determined Euler Poles, the estimates can be improved. However the Euler Poles for the Caribbean and Mariana plates are poorly known, and will require substantial work to be at the proper accuracy for 2022.

As such, within each of the four plate-fixed frames, every point will contain *some* non-Eulerian velocities, but the predominant horizontal signal will have been removed for the majority of each plate. This decision means that coordinates, whether in the global ideal (ITRF or IGS) frame or one of the four terrestrial "plate fixed" frames (of the NSRS) *will* have time dependencies. Those time dependencies will, however, only reflect the deviation of the point's coordinates from the rigid, rotating *frame*. Those deviations, due to non-Eulerian velocities will manifest as velocities within a frame, or "intra-frame velocities" over time, and will be captured in a separate tool, to be discussed later in this report.

5 Ideal frames and plate fixed frames

The use of positioning technologies like GNSS rely upon orbits and/or global tracking stations which are expressed in some ideal frame, such as the IGS14 frame (Rebischung and Schmid, 2016). Such frames do not attempt to minimize horizontal motions on any particular tectonic plate, and thus X, Y and Z (Earth-centered, Earth-fixed or 'ECEF' Cartesian coordinates) are time-dependent in such a frame. This means that latitude and longitude are also time-dependent (as well as ellipsoid heights, though they are driven by horizontal tectonic drift to a much lesser extent). As such, for surveyors or other positioning professionals working on just one plate whose work relies on (preferably) constant horizontal coordinates, the ideal frame is not a preferred choice. Rather, a plate fixed frame can be set up.

Therefore, to sum up, a plate fixed frame can be defined in many ways, but the method chosen for the new terrestrial reference frames will be that two conditions will be met, defining the plate fixed frame in a way that is relatable directly to the ideal frame.

Condition 1: The coordinates of all points in a plate-fixed frame should remain constant through time, provided all of those points rotate about the Euler Pole with the same angular velocity and otherwise have no other motions.[2]

Condition 2: The coordinates of all points in a plate-fixed frame are identical to their coordinates in the ideal frame at some initial chosen epoch t_0.

The use of these two conditions will be presented in the next section to form the mathematical relationship between the ideal frame and any of the four terrestrial "plate fixed" frames of the modernized NSRS.

Expressing the mathematical connection between the ideal frame and a plate fixed frame

As mentioned earlier, positioning could simply be performed entirely in the ideal frame, as long as a user were willing to accept that a coordinate determined on some fixed point at some time will be different than its coordinate at some other time, since all of the tectonic plates have motions within the ideal frame. Thus we can assume that we will always have access to the time-dependent coordinates in the ideal frame, but a mathematical connection must be made to obtain time-dependent coordinates in a plate fixed frame.

Let us begin by presuming that we have an ideal frame, which we call RF1, and whose ECEF coordinates are time-dependent and called (X_1, Y_1, Z_1). See Figure 5.

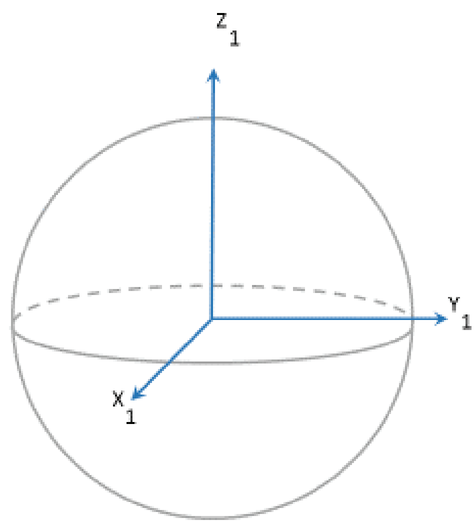


Figure 5: Ideal coordinate frame #1.

[2] We already know that all points have some non-Eulerian motions. This condition therefore can draw the corollary that ***if*** the plate were rigid, ***then*** coordinates in our plate-fixed frame wouldn't change over time.

Also, assume that some rigid tectonic plate sits on the surface of the Earth, and is rotating about an Euler Pole[3]. Assume that we know the co-latitude (θ_0) and longitude (λ_0) of the Euler Pole, in RF1, and also the angular velocity of the tectonic plate about that pole, $\dot{\omega}_0$. . See Figure 6.

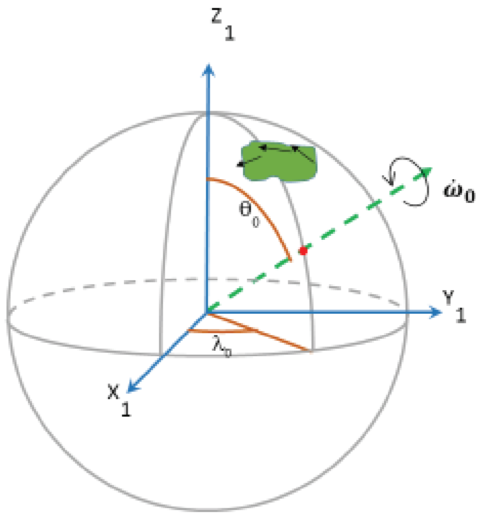
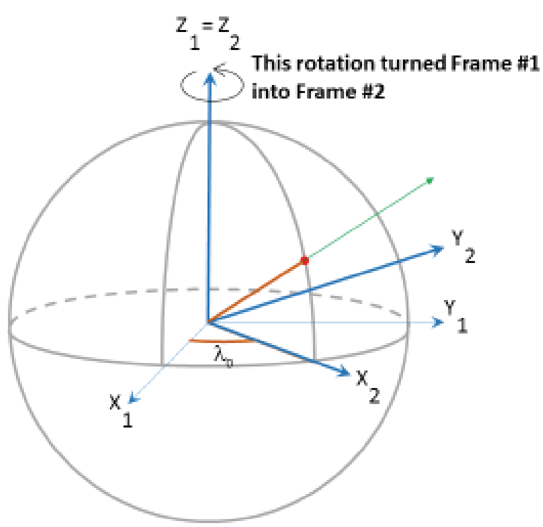


Figure 6: A rotating tectonic plate (green) and its Euler Pole (dashed green arrow and red dot).

We are going to create a new frame, called RF3, the reason for which will become clear soon. To do so, requires first creating an intermediate frame, RF2.

First, let us perform a counter clockwise rotation of ideal frame (RF1) about its Z_1 axis by λ_0, in order to create RF2 where the Euler Pole now lies in the X_2-Z_2 plane.



[3] For simplicity, a spherical Earth will be used in this report. However, the ellipsoidal nature of the Earth does introduce a 2^{nd} order effect and that will be accounted for in the modernized NSRS.

Figure 7: Creating RF2 by rotating RF1 about the Z_1 axis by λ_0.

The mathematical relationship between coordinates in two Cartesian frames (related through a single rotation) is well known and will be presented momentarily. However, before proceeding, a subtle, but critical point should be made: The Euler Pole's location in RF1 (co-latitude and longitude of θ_0 and λ_0) is (for now) presumed to be not moving over time[4]. Therefore, as we write the relationship between coordinates in RF1 and coordinates in RF2, any epoch may be chosen. Therefore (and for reasons that will be clear later))we will explicitly write out two equations; the first for the specific epoch $t=t_0$, and the second for any generic epoch "t".

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t_0} = \begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} = R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \tag{1}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t} = \begin{bmatrix} \cos\lambda_0 & \sin\lambda_0 & 0 \\ -\sin\lambda_0 & \cos\lambda_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t} = R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t} \tag{2}$$

Now proceeding to the creation of RF3, rotate RF2 counterclockwise about its Y_2 axis by θ_0 to establish RF3 which has its Z_3 axis pointing along the Euler pole axis. See Figure 8.

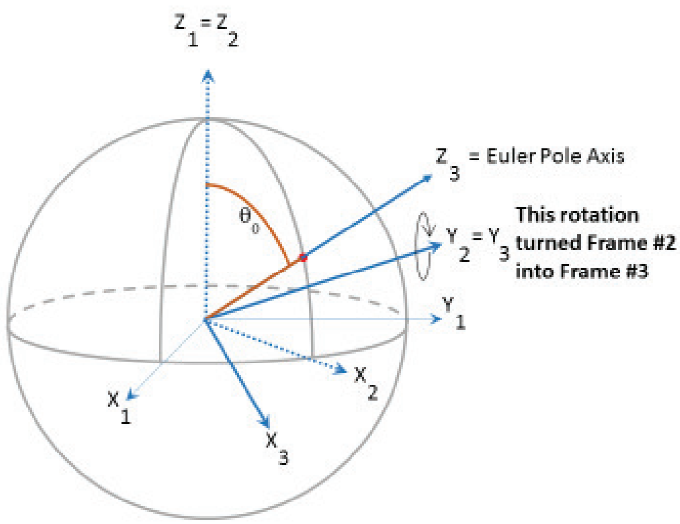


Figure 8: Creating RF3 by rotating RF2 about the Y_2 axis by θ_0.

As before (with RF1 and RF2), we can now write the relationship between RF1 coordinates and RF3 coordinates at any epoch, since the Euler Pole isn't moving.

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[4] Like any modeled quantity, there is uncertainty not only in the Euler Pole's location but possibly in its stability within the ideal frame itself. Any such uncertainty or instability will be estimated by NGS and will propagate into the coordinates and uncertainties in the four terrestrial reference frames.

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_{t_0} = \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t_0} = R_2^{\theta_0} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t_0} = R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \tag{3}$$

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_{t} = \begin{bmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t} = R_2^{\theta_0} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}_{t} = R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t} \tag{4}$$

To be explicit: RF1, RF2 and RF3 all have a fixed orientation to one another over time. These frames do not rotate over time. However, a point sitting on a rigid tectonic plate, rotating about the Euler Pole will have time-dependent coordinates in all three frames. It just so happens that the computation of that time dependency, as described below, is much simpler in RF than in the other two, which is why RF3 was introduced.

Since RF3 has its Z_3 axis aligned with the Euler Pole, then the time-dependent RF3 coordinates (X_3, Y_3, Z_3) of a point sitting on a plate which rotates about the Euler pole may very easily be computed simply by applying a rotation about the Z_3 axis to those coordinates. First, assume the time elapsed since epoch t_0 (when the ideal and plate-fixed frames were aligned) is Δt, where $\Delta t = t - t_0$. Then, assume the angular velocity of the plate rotation about the Euler Pole is $\dot{\omega}_0$ (in, say, milli-arcseconds per year). Thus, in the time interval between t_0 and t, the plate rotated by an angle "α" about the Euler Pole (or, equivalently, about the Z_3 axis) where $\alpha = \dot{\omega}_0 \Delta t$. In order to visualize this, let us view frame #3 from the perspective that the Z_3 axis points upwards, and we can see our continent. Let us then identify some point on that continent. See Figure 9:

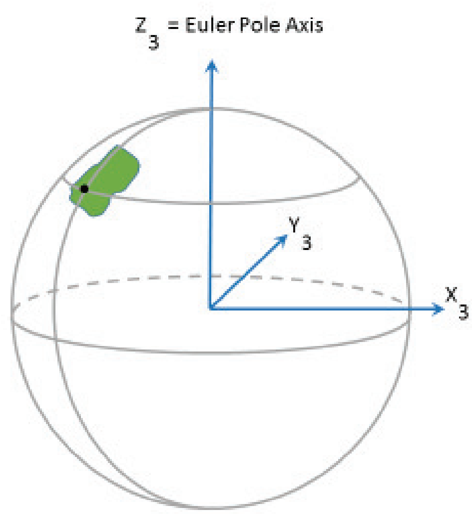

Figure 9: New perspective of RF3. Dot (black) is any point on the tectonic plate at t_0.

Now let us show the motion of the point on the tectonic plate by plotting its location at t_0 and t. See Figure 10.

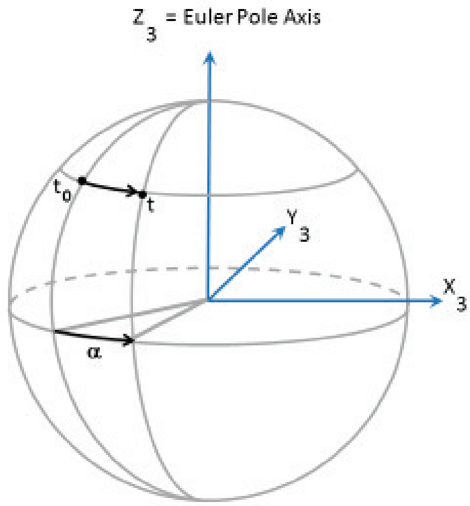

Figure 10: The simple motion of any point over time on the rotating continent, when seen in RF3.

Then, the relationship between (X_3, Y_3, Z_3) at time t and (X_3, Y_3, Z_3) at time t_0 is just a rotation about the Z_3 axis by an angle of α:

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_t = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_{t_0} = R_3^{\alpha} \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_{t_0} \tag{5}$$

Note, that this rotation is not creating a new frame, but is expressly defining the Euler-Pole motion (time-dependence) of a point's coordinates *within* RF3. For this reason, the rotation matrix in equation 5 is the *inverse* of the standard rotation matrix about a Z axis. This represents the difference between:

rotating a frame about its Z axis, and computing the effect on an unmoving point

and

keeping the frame unmoving, while *rotating a point* about the frame's Z axis.

The former type of change was seen in equations 1 and 2. The latter type of change is seen in equation 5.

See that in equation 5, unlike equations 1 through 4, the epoch on the left hand side (t) is different from the epoch on the right hand side (t_0). Now, invoking equation #3 and applying it to equation #5 allows us to express the time-dependent RF3 coordinates in terms of coordinates at t_0 in RF2 but more importantly in the ideal frame, RF1:

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_t = R_3^{\alpha} \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_{t_0} = R_3^{\alpha} R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \tag{6}$$

However, repeating equation 4 so it can immediately be compared it to equation 6:

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}_t = R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \qquad (7)$$

Note that equations 6 and 7 have the same left hand side (time-dependent coordinates in RF3). As such, let us set their right hand sides equal to one another:

$$R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = R_3^{\alpha} R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \qquad (8)$$

Re-arranging equation 8 yields:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = \left[R_1^{\lambda_0}\right]^{-1} \left[R_2^{\theta_0}\right]^{-1} R_3^{\alpha} R_2^{\theta_0} R_1^{\lambda_0} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \qquad (9)$$

Equation 9 shows the relationship between RF1 coordinates over time and RF1 coordinates at epoch t_0 (note its much more complicated nature than the frame 3 relationship from equation 5). In other words, this is defining the Euler-Pole motion (time-dependence) of a point's coordinates *within* the ideal frame (RF1). The right hand side, reading from right to left, may be interpreted as "start with RF1 coordinates at t_0, rotate into RF3, then let coordinates change over time Δt through the plate rotation, then rotate back to RF1".

For the sake of brevity, combine the 5-rotation matrices on the right hand side of equation 9 into one matrix called "M":

$$M = \left[R_1^{\lambda_0}\right]^{-1} \left[R_2^{\theta_0}\right]^{-1} R_3^{\alpha} R_2^{\theta_0} R_1^{\lambda_0} \qquad (10)$$

Where it should be remembered that M is dependent upon θ_0, λ_0, and α (or $\dot{\omega}_0$ and Δt):

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = M \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \qquad (11)$$

Turning our attention now to the plate-fixed frame, let us refer to it with lower case letters (x, y, z). Earlier, we defined that "plate fixed" in the modernized NSRS, as each tectonic plate having one "plate fixed" terrestrial reference frame, where *the grid of parallels and meridians itself will be rigid*, and rotate about the best computable Euler Pole for that plate (computed after accounting for, and removing, any other spurious horizontal motions). To express this definition mathematically, two conditions were introduced and will now be invoked.

The definition of "plate fixed" is expressed in independent equations, each of which fulfills one of the conditions mentioned earlier. The first states, in brief, that "in any given plate fixed frame, the plate fixed coordinates do not change over time" (Condition 1 above). Thus:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} \qquad (12)$$

Before proceeding, it is critical to remember that equation 12 is only valid for a point whose entire motion (in the ideal frame) is that of rotation about the Euler Pole (i.e. it's true only if the plate is as rigid as the grid of parallels and meridians being laid over it and there is no vertical motion at all). The assumptions are likely not true in the real world, as *all* points are expected to have *some* intra-frame motion not fully described by the plate's rotation (whatever their scale in time or space). As such, we expand equation 12 to reflect this fact:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} + \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}_{\Delta t} \qquad (13)$$

Equation 13 holds now for any point in our plate fixed reference system, the point doesn't even have to be located on the plate to which the Euler Pole refers. For example, one can define "North American Plate Fixed coordinates" for a point on the Pacific Plate, because the motion of the Pacific Plate relative to the North American Plate, can be accounted for in the "intra-frame velocity" vector.

Because the dx, dy and dz motions can have many different scales in both time and space, no further attempt to clarify them is made here except to note that they translate into 3-dimensional intra-frame velocities (including changes to latitude and longitude that aren't captured by the plate rotation model and the ellipsoid height velocity signal). They will be carried forward in the following derivations and discussed later. But remember that, for most points on the so-called "rigid" part of the plate, the dx, dy, dz vector are expected to be exhibit horizontal motions somewhere between "small" (1-2 mm / year) and "zero", relative to the magnitude of the plate rotation (1- 3 cm / year). Vertical motions may be significantly larger than this in any part of the plate that is experiencing rapid subsidence or uplift. Naturally, these magnitudes exclude those parts of the plate that are undergoing significant deformation (such as Southern California for the North American Plate).

Equation 13 showed equivalence (and equation 12 showed dependence) over time of the plate-fixed coordinates to some chosen set of plate-fixed coordinates at some particular epoch t_0, but does not state what the actual plate-fixed coordinates *are* at that epoch. That bring us to the second plate-fixed condition which states that "the plate fixed coordinates at epoch t_0 are equal to the ideal frame coordinates at that same epoch" (Condition 2). Mathematically:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \quad (14)$$

Note that equation 14 does not suffer from the issue of rigid versus non-rigid location of points. It simply sets all coordinates in the plate fixed frame equal to those in the ideal frame, without any regard for where, on the plate, such a point sits; it gives us an initialized set of plate-fixed coordinates.

Equations 11-14 are used to derive the relationship between plate-fixed coordinates over time (which is the desired quantity) and ideal frame coordinates over time (which is usually the quantity first computed when using GNSS). Beginning with equation 11, and then invoking equations 14 and then 13, one can see the following:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = M \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} = M \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} = M \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix}_t - \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}_{\Delta t} \right\} \qquad (15)$$

Solving equation 15 for the time-dependent plate-fixed coordinates yields:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = M^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t + \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}_{\Delta t} \qquad (16)$$

What equation 16 states is that, if there no intra-frame motions, then a simple rotation matrix, M^{-1}, provides the connection between time-dependent ideal frame coordinates (which are usually output by a GNSS software package) and the time-dependent plate-fixed coordinates (which are often desired by geospatial professionals working on that plate). What is not obvious from equation 16 is that, in the absence of intra-frame motions, time-dependent plate-fixed coordinates are constant over time (the desired outcome of adopting a plate-fixed reference system). The derivation of this fact is presented below before proceeding.

Begin by modifying equation 16 so that there are no intra-frame motions:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = M^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \qquad (17)$$

Our goal is to show that, the left hand side of equation 17 is actually time-*independent* for any point on the tectonic plate that is rotating about our given Euler Pole at the set rate of rotation of that plate with no intra-frame motions. Begin by expanding the right hand side of equation 17, using equation 11:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = M^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = M^{-1} M \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_{t_0} \qquad (18)$$

Thus we can see that without intra-frame motions (assuming a rigid, rotating plate without any vertical signals), the "plate fixed" coordinates, expressed as a function of time, do not deviate; they are fixed at their initial values, as set at epoch t_0 (see equation 14).

6 The 2022 Reference Frames

The National Geodetic Survey, in preparing for the 2022 replacement of the NAD 83 frames, received user feedback through multiple channels (including two National Geospatial Summits, in 2010 and 2015). In 2016, reflecting on that user feedback and considering the appropriate balance of science and stewardship, NGS held a number of internal discussions to rigorously define the new geometric reference frame approach for 2022. The result of those discussions can be summarized as follows:

1) In 2022, the NSRS will contain four newly defined terrestrial reference frames, one for each of these four tectonic plates: North American, Pacific, Mariana and Caribbean.
2) The definitional relationship between the latest IGS frame and each of the four terrestrial frames will adhere to Conditions 1 and 2 from earlier.

The intra-frame velocities *will not* be removed when NGS provides coordinates in the new reference frames. Instead, they will be provided as a separate service by NGS as described below.

NGS can, with a great deal of accuracy, provide users the ability to position themselves, at time "t" in the ideal frame. NGS also knows that, with similar accuracy, the plate rotations of the North American and Pacific plates can be computed and removed, providing accurate positions in the "plate fixed frame" at time "t".[5] Therefore, NGS will, define four plate fixed terrestrial reference frames, each related to the ideal (IGS) frame through a simple plate rotation model. Coordinates in each frame will be time-dependent because any intra-frame velocities which points are experiencing will change the point's coordinates in the plate fixed frame over time. However, NGS will also model those intra-frame velocities and provide that model as a method for users to compare points at common epochs. The level of accuracy of the intra-frame velocity (IFV) model remains TBD, but it will vary as a function of geophysical complexity and available geodetic control.

Sustaining the accuracy of the IFV model grows increasingly difficult if the goal is to model every intra-frame motion of every point on each continent through all time. Even from a horizontal-only perspective, the task is daunting, as every earthquake, compression, GIA signal, coastal sloughing or other geophysical signal, in all scales of time and space would need to be completely and accurately modeled. The situation is further complicated with the inclusion of the vertical, which has significantly more localized signals than the horizontal. In an effort to be fiscally responsible, NGS intends to provide a service that can deliver the highest achievable levels of accuracy without attempting to model the IFVs an unwieldy and unsustainably complex continent-wide deformation model that is in constant flux.

This is not to say that intra-frame motions are not important or that they will not be provided. But the terrestrial ("plate fixed") reference frames themselves will only be related to the ideal frame through the rotation of the plate.

To re-iterate, and to repeat equation 17: By definition, each of the four terrestrial reference frames will have their time-dependent coordinates defined through a rotation matrix, M, in relation to the time-dependent coordinates in the ideal (IGS) frame :

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = M^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \qquad (19)$$

There will be one unique 3x3 "M" matrix determined for each plate, and the ideal frame #1 will be the most recent version of the IGS frame available by 2022. The epoch "t_0" remains to be chosen, but will be identical for all four frames. Furthermore, while the determination of a plate's Euler Pole and rotation rate are much easier today with decades of GPS data to work with, it is not a perfect process. As mentioned earlier, the current knowledge of the Caribbean and Mariana plates is fairly weak.

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5 The current knowledge of the Caribbean and Mariana plate rotations is much weaker than the North American and Pacific plates, and NGS will strive to fix that situation before 2022.

Therefore, NGS will likely need to re-evaluate these determinations about every decade, and possibly update any of the four frames as needed, to ensure the frame and the plate are rotating as congruently as possible. As such, NGS will be meticulous in providing a "version number" of each update to a frame, as well as metadata about what changes occur with any new version.

Note that the epoch on the left hand side of equation 19 is the epoch of the survey "t". Previous incarnations of the reference frames of the National Spatial Reference System have attempted to connect frame #1 coordinates at "t" with unchanging coordinates at some chosen reference epoch "t_0". This is no longer the approach for the primary service NGS will provide. Consideration of the time dependency of a point's "plate fixed" coordinates, when it is well known that such a frame relies on the unrealistic assumption that the tectonic plate is "rigid," will allow users to observe the intra-frame motions associated with "time-dependent plate fixed coordinates.". As a secondary service, NGS will provide a model of the intra-frame velocities (IFV) so that users may estimate the change in coordinates for any particular point at disparate survey epochs.

Users of the NSRS in stable (rigid) parts of a plate may expect to see small (to negligible) intra-frame velocities. If NGS determines that a point's intra-frame velocities are measurably zero, through either repeat surveys or through an IFV model, that information will be provided. NGS will provide the intra-frame velocities on all points, even on points when the observed or modeled magnitudes of those velocities are zero.

7 14-Parameter Transformation between IGS and four *TRF2022's

It will be instructive to actually derive the transformation between IGS and the four *TRF2022's from equation 19. However, this transformation will, of necessity, diverge slightly from the common form of a 14 parameter Helmert transformation due to the treatment of epochs when converting from IGS to the four TRFs of 2022. By way of explanation, consider the form of equation 19, which has, on either side of the equals sign, coordinates in two different frames but at the same epoch, "t". In general terms:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t \xleftrightarrow[M^{-1}\ or\ M\ depending\ on\ direction]{} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \qquad (20)$$

Contrast this with the common form of a 14 parameter transformation (Soler and Marshall, 2003; equation 3) which has coordinates, but no velocities, in one frame at a *reference* epoch "t_0" while coordinates *and* velocities of those same points are in the second frame at *survey* epoch "t":

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} \xleftrightarrow[14\ Parameter\ Transformation]{} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ \dot{X}_1 \\ \dot{Y}_1 \\ \dot{Z}_1 \end{bmatrix}_t \qquad (21)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{t_0} \xleftrightarrow[14\ Parameter\ Transformation]{} \begin{bmatrix} X_1 + (t - t_0)\dot{X}_1 \\ Y_1 + (t - t_0)\dot{Y}_1 \\ Z_1 + (t - t_0)\dot{Z}_1 \end{bmatrix}_t \qquad (21)$$

The structure of these two relationships is different, as are their goals. The goal of applying the Euler Pole rotation (M^{-1} matrix) to the IGS frame in equation 20 (or equation 19) is not to arrive at *TRF2022 coordinates at a *reference* epoch, but to arrive in that TRF at the *same* epoch as the IGS frame. ***Thus a 1-to-1 correspondence between a standard 14 parameter transformation and equation 19 cannot be drawn***.

However, with a few modifications, equation 19 can be equated to a modified 14 parameter transformation. For example, Stanaway, et al. (2014), claim that a simple 3-parameter transformation can be developed which will effectively apply the relationship seen in equation 19, where those three parameters are rotation rates about the three axes of the ITRF frame. This is not terribly surprising since there are, in fact, three parameters in equation 19: the two Euler Pole coordinates and the rotation about that pole. But geodesists tend to prefer applying parameters only as translations, rotations about the ideal frame axes and scale parameters. Such a transformation from the three parameters in equation 19 to three Cartesian axial rotation rates is not trivial without adopting the "small angle approximation", at which point the derivation becomes much easier.

Beginning with a quick refresher on Helmert Transformations, recall the general form for any 7-parameter transformation. The Bursa-Wolf version (Rapp, 1989), will be adopted (dropping the subscript "1" from the variables X, Y and Z):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + (1+s) R_Z(\omega_Z) R_Y(\omega_Y) R_X(\omega_X) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{22}$$

where:

$$R_X(\omega_X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_X & \sin\omega_X \\ 0 & -\sin\omega_X & \cos\omega_X \end{bmatrix} \tag{23}$$

$$R_Y(\omega_Y) = \begin{bmatrix} \cos\omega_Y & 0 & -\sin\omega_Y \\ 0 & 1 & 0 \\ \sin\omega_Y & 0 & \cos\omega_Y \end{bmatrix} \tag{24}$$

$$R_Z(\omega_Z) = \begin{bmatrix} \cos\omega_Z & \sin\omega_Z & 0 \\ -\sin\omega_Z & \cos\omega_Z & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{25}$$

These rotation matrices are consistent with a positive rotation in the counterclockwise direction of a right-handed coordinate system, when viewed down the axis from the viewpoint of its positive end (Leick and van Gelder, 1975).

There are many variations on equation 22, for example with the scale factor (1+s) applied after the transformation vector is applied, or with the scale factor written "(1-s)", or with the rotations positive clockwise, rather than counterclockwise. There is no right or wrong form of these equations, but it is imperative that one clarify which version is being used for which application.

In order to create a 14, rather than 7, parameter transformation, one need only make each of the 7 parameters time-dependent. However, if *both* Cartesian triads are also made time-dependent this will create an *alternative* 14 parameter transformation, of a slightly different nature than that provided in equation 21:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} T_x(t) \\ T_y(t) \\ T_z(t) \end{bmatrix} + (1 + s(t)) R_Z(\omega_Z(t)) R_Y(\omega_Y(t)) R_X(\omega_X(t)) \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} \quad (26)$$

where:

$$\begin{bmatrix} T_x(t) \\ T_y(t) \\ T_z(t) \\ s(t) \\ \omega_X(t) \\ \omega_Y(t) \\ \omega_Z(t) \end{bmatrix} = \begin{bmatrix} T_x(t_0) + (t - t_0)\dot{T}_x \\ T_y(t_0) + (t - t_0)\dot{T}_y \\ T_z(t_0) + (t - t_0)\dot{T}_z \\ s(t_0) + (t - t_0)\dot{s} \\ \omega_X(t_0) + (t - t_0)\dot{\omega}_X \\ \omega_Y(t_0) + (t - t_0)\dot{\omega}_Y \\ \omega_Z(t_0) + (t - t_0)\dot{\omega}_Z \end{bmatrix} \quad (27)$$

For simplicity, combine the three rotation matrices into one:

$$R_{ZYX}(t) = R_Z(\omega_Z(t)) R_Y(\omega_Y(t)) R_X(\omega_X(t)) \quad (28)$$

so that:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} T_x(t) \\ T_y(t) \\ T_z(t) \end{bmatrix} + (1 + s(t)) R_{ZYX}(t) \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} \quad (29)$$

Now, if equation 29 is compared to equation 19, a few things become immediately obvious:

1) There is no translational vector in equation 19, so the time-dependent translation vector in 29 must be zero, and thus 6 of the 14 parameters are zero:

$$\begin{bmatrix} T_x(t) \\ T_y(t) \\ T_z(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (30)$$

or

$$\begin{bmatrix} T_x(t_0) \\ T_y(t_0) \\ T_z(t_0) \\ \dot{T}_x \\ \dot{T}_y \\ \dot{T}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (31)$$

2) There is no scale factor in equation 19, and so the time-dependent scale factor in equation 29 must be zero, and thus two more of the 14 parameters are zero:

$$s(t) = 0 \quad (32)$$

or

$$\begin{bmatrix} s(t_0) \\ \dot{s} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (33)$$

3) Rotation matrix "M^{-1}" must therefore be identical to rotation matrix R_{ZYX}. As such, it should be possible to equate the time-dependent axial rotation angles, $\omega_X(t)$, $\omega_Y(t)$ and $\omega_Z(t)$ to the fixed angles of θ_0, λ_0 and the time-dependent angle $\alpha(t)$ (or its components $\dot{\omega}_0 \bullet \Delta t$). That is:

$$M^{-1} = R_{ZYX} \quad (34)$$

Without some simplifying approximations, the relationship implied by conclusion #3 above is more difficult to derive than equation 34 would imply. This is because both the M^{-1} and R_{ZYX} matrices are fairly complicated. Some simplifications can be made to help solve the problem. The first is that the angle $\alpha(t)$ will be "small". To apply that approximation, first let's express the exact formulation for the M^{-1} matrix, which can easily be inferred from equation 10 by noting that all five component matrices of "M" are invertible:

$$M^{-1} = \left[R_1^{\lambda_0}\right]^{-1} \left[R_2^{\theta_0}\right]^{-1} [R_3^{\alpha}]^{-1} R_2^{\theta_0} R_1^{\lambda_0} \quad (35)$$

The $[R_3^{\alpha}]^{-1}$ matrix is:

$$[R_3^{\alpha}]^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (36)$$

Then, these small angle assumptions can be made:

$$\cos(\alpha(t)) \to 1 \quad (37)$$

$$\sin(\alpha(t)) \to \alpha(t) \quad (38)$$

Applying equations 37 and 38 to 36 yields:

$$[\widetilde{R_3^{\alpha}}]^{-1} = \begin{bmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = (I + A) \qquad (39)$$

Where the tilde is used to indicate "approximation". The reason for splitting the matrix into *I* and *A* components will be obvious soon.

Applying equation 39 to 35:

$$\widetilde{M^{-1}} = \left[R_1^{\lambda_0}\right]^{-1}\left[R_2^{\theta_0}\right]^{-1}[\widetilde{R_3^{\alpha}}]^{-1}R_2^{\theta_0}R_1^{\lambda_0} = \left[R_1^{\lambda_0}\right]^{-1}\left[R_2^{\theta_0}\right]^{-1}(I+A)R_2^{\theta_0}R_1^{\lambda_0}$$

$$= \left[R_1^{\lambda_0}\right]^{-1}\left[R_2^{\theta_0}\right]^{-1}(I)R_2^{\theta_0}R_1^{\lambda_0} + \left[R_1^{\lambda_0}\right]^{-1}\left[R_2^{\theta_0}\right]^{-1}(A)R_2^{\theta_0}R_1^{\lambda_0}$$

$$= I + \alpha(t)\begin{bmatrix} 0 & cos\theta_0 & -sin\lambda_0 sin\theta_0 \\ -cos\theta_0 & 0 & cos\lambda_0 sin\theta_0 \\ sin\lambda_0 sin\theta_0 & -cos\lambda_0 sin\theta_0 & 0 \end{bmatrix} \qquad (40)$$

See now that by splitting into I and A, the *I* portion of the equation collapses into another I, while the *A* component collapses into a simple skew symmetric matrix.

Acknowledging that the effect of the total rotation, $\alpha(t)$ must be split into rotations among the three axes of the ideal frame, and since $\alpha(t)$ is "small", it can be concluded that the axial rotations must also be small. Thus, matrix R_{ZYX} reduces to:

$$\widetilde{R_{ZYX}} = \begin{bmatrix} 1 & \omega_Z & -\omega_Y \\ -\omega_Z & 1 & \omega_X \\ \omega_Y & -\omega_X & 1 \end{bmatrix} \qquad (41)$$

Now equate the approximations of M^{-1} and R_{ZYX} to one another (applying equations 39 and 40 to equation 34):

$$I + \alpha(t)\begin{bmatrix} 0 & cos\theta_0 & -sin\lambda_0 sin\theta_0 \\ -cos\theta_0 & 0 & cos\lambda_0 sin\theta_0 \\ sin\lambda_0 sin\theta_0 & -cos\lambda_0 sin\theta_0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \omega_Z & -\omega_Y \\ -\omega_Z & 1 & \omega_X \\ \omega_Y & -\omega_X & 1 \end{bmatrix} \quad (42)$$

Equation 42 allows for an easy solution to the three axial rotations in terms of the Euler Pole's location and angular velocity:

$$\omega_X = \alpha(t)cos\lambda_0 sin\theta_0 \qquad (43)$$

$$\omega_Y = \alpha(t)sin\lambda_0 sin\theta_0 \qquad (44)$$

$$\omega_Z = \alpha(t)cos\theta_0 \qquad (45)$$

Recall, however, that the ω_X, ω_Y and ω_Z values are time-dependent (see equation 27). Applying equation 27 and also applying the expansion of $\alpha(t)$ into its components, yields:

$$\omega_X(t_0) + (\Delta t)\dot{\omega}_X = [\dot{\omega}_0 \Delta t]cos\lambda_0 sin\theta_0 \qquad (46)$$

$$\omega_Y(t_0) + (\Delta t)\dot{\omega}_Y = [\dot{\omega}_0 \Delta t]sin\lambda_0 sin\theta_0 \qquad (47)$$

$$\omega_Z(t_0) + (\Delta t)\dot{\omega}_Z = [\dot{\omega}_0 \Delta t]cos\theta_0 \qquad (48)$$

The first term on the left hand side of equations 46, 47 and 48 are all constants. There is no corresponding constant value on the right hand side of those equations. For the purposes of convenience, it would be best to invoke Condition #2 from earlier, which means that there should be no constant difference between the plate-fixed frame and the ideal frame. Thus the constant terms on the left hand side of equations 46-48 should be set to zero. As such, there is no constant rotation about the ideal frame axes present in equation 19. That is:

$$\begin{bmatrix} \omega_X(t_0) \\ \omega_Y(t_0) \\ \omega_Z(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (49)$$

Thus, equations 46-48 simplify to:

$$(\Delta t)\dot{\omega}_X = [\dot{\omega}_0 \Delta t] cos\lambda_0 sin\theta_0 \qquad (50)$$

$$(\Delta t)\dot{\omega}_Y = [\dot{\omega}_0 \Delta t] sin\lambda_0 sin\theta_0 \qquad (51)$$

$$(\Delta t)\dot{\omega}_Z = [\dot{\omega}_0 \Delta t] cos\theta_0 \qquad (52)$$

Dividing by the common term, Δt, on both sides of equations 50-52, they also simplify further. Applying this simplification, and returning to the use of the "X_1" "Y_1" and "Z_1" terminology yields the relationship between the rotation rates (radians/year) around the three ideal frame axis ($\dot{\omega}_{X_1}, \dot{\omega}_{Y_1}, \dot{\omega}_{Z_1}$) and the co-latitude and longitude of the Euler Pole in the ideal frame (θ_0, λ_0) and the rotation rate (radians/year) of the plate about the Euler Pole ($\dot{\omega}_0$).

$$\dot{\omega}_{X_1} = \dot{\omega}_0 cos\lambda_0 sin\theta_0 \qquad (53)$$

$$\dot{\omega}_{Y_1} = \dot{\omega}_0 sin\lambda_0 sin\theta_0 \qquad (54)$$

$$\dot{\omega}_{Z_1} = \dot{\omega}_0 cos\theta_0 \qquad (55)$$

Inverting equations 53-55 yields the following relationships:

$$\theta_0 = \frac{\pi}{2} - ArcTan\left[\frac{\dot{\omega}_{Z_1}}{\sqrt{\left(\dot{\omega}_{X_1}^2 + \dot{\omega}_{Y_1}^2\right)}}\right] \qquad (56)$$

$$\lambda_0 = ArcTan\left[\frac{\dot{\omega}_{Y_1}}{\dot{\omega}_{X_1}}\right] \qquad (57)$$

$$\dot{\omega}_0 = \sqrt{\dot{\omega}_{X_1}^2 + \dot{\omega}_{Y_1}^2 + \dot{\omega}_{Z_1}^2} \qquad (58)$$

These relationships are identical (but for the difference in latitude vs co-latitude) to those expressed by Stanaway et al (2014). Similar equations are given in Ali Gourdarzi et al (2014).

In summary, the official relationship between IGS and the four TRFs of 2022 is expressed in equation 19. However, that relationship can be transformed, using small angle approximations, to a slightly modified 14 parameter transformation (equation 26) where time dependency exists on both sides of the equation. Of the 14 parameters in equation 26, eleven are zero. The remaining three are the rotation rates (radians/year) around the three ideal frame axes, as expressed in equations 53-55.

8 Intra-frame 3-D Velocities in 2022

Of all the components of this Blueprint document, this section is the least "final". That is, NGS has shown, in a fairly finalized form, how it will provide time dependent plate-fixed coordinates in the modernized NSRS. And NGS knows it will be providing a service to relate such coordinates through time, with an emphasis on comparing coordinates at epochs of convenience, by modeling intra-frame velocities. But many of the details about how such services are actually going to look remain under development. Therefore, readers are cautioned to view the following section in that light.

With that in mind, there are many ways to determine velocities, besides GIA models or plate rotation models. Directly measuring movements of points can be categorized as geokinematics (basically "determination of things relative to Earth in space and through time"), and it is highly data driven. Contrast that with the field of geodynamics which attempts to model geophysical processes and express the motion of the crust through a more mechanistic method. Both approaches have advantages and disadvantages, and the adoption of one over the other depends on data availability, accuracy requirements, and intended applications of the end user. This section will address the NGS approach to determining intra-frame 3-D velocities.

To reiterate, in the four new terrestrial reference frames of 2022, every active or passive geodetic control point is expected to have some intra-frame 3-D velocity. With the tectonic plate rotation removed, the dominant *horizontal* signal on the *majority* of the plate should be gone, leaving small horizontal intra-frame motions in those regions. But GIA, subsidence and the parts of the plate that are not rigid and/or not rotating at the plate's computed rate, will result in intra-frame motions that are not small.

When the Euler Pole is computed for each of the four terrestrial reference frames, it will be "uncorrupted by GIA". While GIA is mostly a vertical signal, it does have a horizontal component, and that horizontal component will be separated from the plate rotation itself, so that the Euler Pole only reflects actual rotation of the (not so rigid) plate.

Additionally, while horizontal velocities will be separated into "Euler Pole Rotations" and "intra-frame velocities" (including the horizontal GIA signal), all vertical velocities will fall into the category of "intra-frame velocities" since the horizontal Euler Pole rotation has no vertical manifestation.

Historically, NGS has provided a model of horizontal motions (both plate rotational velocities and horizontal intra-frame velocities) through the Horizontal Time Dependent Positioning (HTDP) computer program. However, HTDP has never supported vertical velocities, except in central Alaska.

The general purpose of HTDP in the past has been to provide a method by which two surveys of the same GNSS vector (baseline between two points) might be compared, when they were performed at different time epochs. That approach supported the philosophy that geodetic control should be provided at a single reference epoch: that each point should have a singular set of fixed coordinates, and that multiple surveys before or after that epoch could have their vectors "moved through time" to support the creation of a consistent coordinate set on that point. Thus, multiple surveys, each showing unique location information on a point, would have that vast quantity of information reduced to a singular coordinate set. This required that HTDP provide geodetic quality models of temporal movements at control points.

To provide such a service, HTDP relied on geophysical models of crustal dynamics including some compressions and earthquakes. That is, aside from using actual geodetic measurements at geodetic control points, additional information (models of the entire crust in several western states and Alaska) were necessary to support the proper functioning of HTDP. Failure to completely model a seismic event, for example, meant that HTDP could not fully model (at geodetic accuracies) the horizontal motion at geodetic control points. Further, HTDP includes no model of vertical motion at all (in most areas) and most of the data coming to NGS for the creation of HTDP came from disparate external sources, such as universities.

NGS will adopt a different approach in 2022. Because geodetic control is mostly about knowing where geodetic control points *are* (and to a lesser extent, about knowing where they were and predicting where they may be in the future), it is not necessary to maintain models about the *entire crust* to perform this essential function. A new survey on a passive control point yields new information about where that point is. If those new coordinates are computed in the same frame as previously determined coordinates, then the difference in the two coordinate sets is direct evidence of errors in one or the other survey, differences in data quality, differences in processing strategy or possibly actual movement. There is no compelling reason why a geophysical model is needed to compare the two sets of coordinates, though such models may be used to attempt to *explain* the cause observed difference. NGS does not view the explanation of why two different geodetic quality surveys yield different coordinates at different times as a mission-essential function of the NSRS. Chasing down the "why" of such changes is a serious drain of resources without accomplishing the goal of geodetic control itself. Consider: If such a model fails to explain the difference in coordinates, what can be concluded? Possibly that some error was made in one survey or the other (or both); possibly that some geophysical motion was not properly accounted for in the model; possibly both of these, or neither. The point is, from a geodetic control standpoint, each survey showed where the passive control was at the time of the survey, and such knowledge of its position was good and useful information for some indeterminate time afterwards. Nonetheless, there is value in providing some service, with a low cost/benefit ratio, which can attempt to describe the actual motion of the point through time, even if such a service does not attempt to explain *why* the motion occurred. Such a model of intra-frame velocities (IFV) will be provided by NGS as an intra-frame velocity model (IFVM), but the exact nature of it remains to be determined.

The continuous monitoring of *active* control points (CORS) yields continuous information about where they are as well as their history. As stated in the NGS Ten Year Strategic Plan (2013-2023), the primary access points to the new terrestrial reference frames will be through CORS. Passive control will be reduced in function to a secondary access.

Passive control will be useful for monitoring change in the new frames. This is different from the current philosophy which presumes that NGS will model change through HTDP, and that new surveys will continue to match old, epoch-specific coordinates by applying HTDP or by making adjustments to coordinates from passive marks that have moved since a past epoch but are defined as fixed. To summarize: To test the velocity models, if repeated surveys occur, NGS will use those repeated surveys at passive control points to yield the history of 3-D coordinates on such points (e.g. for comparison against a CORS-based IFVM rather than presuming to model such a history using geophysical models. However, the overwhelming majority of points with survey data in the NGS archives have been surveyed only a single time in their history. This obviously does not allow for any monitoring of those points at this time.

The absence of repeated occupations on most passive control means that such points will have coordinates so old that they may not be reliable. In order to help understand the potential movements of points, a CORS-data-driven velocity model will be available from NGS. While such a model breaks down in areas of significant localized intra-frame motions, or lack of CORS coverage, it is nonetheless a simple model to produce and is easily set up in production mode, by (for example) gridding CORS velocities and discontinuities through their history, and completing a 3D interpolation of the grid between CORS stations. With no reliance upon external geophysical models, such a model will be easy to produce from validated, in-house CORS data. This is not to say that no other 3-D intra-frame velocity models will be available through NGS products. It only means that the CORS data-driven velocity model will be the first and easiest to produce.

However, if the purpose of geodetic control is to provide knowledge of where a point is, based on geodetic data collection at that point, what would be the purpose of such a 3-D intra-frame velocity model? The purpose of such modeling is that it will significantly assist the *engineering* and *mapping* community (a much larger user base then the geodetic surveying community) by providing coordinates (at mapping-level accuracies) at a common epoch. Thus, positions at different epochs can all be compared at a single "epoch of convenience". Unlike the gridded velocity component of HTDP, which was used as a way to move multiple geodetic surveys through time so that a geodetic quality coordinate might be stated as the target, this intra-frame motion model will be used to move multiple geodetic quality coordinates through time and produce *mapping-accuracy* coordinates.

In many ways, this philosophy is very similar to coordinate transformation software like NADCON. NGS has stated, since the inception of NADCON, that using a model to transform a map or survey from one datum to another is not equivalent to re-adjusting the original observations to new geodetic control. Similarly, a model of crustal motion which attempts to move a coordinate from one point in time to another is not the same as actually performing a geodetic survey at the target epoch. As such, NGS views the coordinates coming from such a "temporal transformation"[6] as not accurate enough to be called "geodetic control". Nonetheless, such temporally transformed coordinates can be used to:

A) Move a map from one epoch to another
B) Produce coordinate transformation software between epochs
C) Move survey positions from one epoch to another, at the cost of a loss of accuracy

Therefore, NGS currently is investigating how to provide an intra-frame 3-D velocity model that is driven by CORS data directly to express all velocities left over after the removal of horizontal plate rotation, in three dimensions. This will allow NGS to provide epoch-specific <u>*mapping-accuracy coordinates*</u> at passive control for the purposes of transformations and other non-geodetic-quality uses of NGS data. Such coordinates will likely be updated every 5 or 10 years; such an interval is yet to be determined.

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[6] As opposed to NADCON which would be called a "datum transformation"

However, the creation of an NGS intra-frame velocity model does not preclude other models from being created and applied to the time-dependent coordinates in the four new plate-fixed terrestrial reference frames of the modernized NSRS. NGS will support interactions of such models with time-dependent NSRS coordinates, but NGS is not currently planning to provide a service to "move data through time and then adjust it all together at a common epoch" as is the current methodology. Rather, OPUS products and services will only yield adjusted coordinates at survey epochs as the primary service, and then NGS will apply IFV models to relate those coordinates to others at a common epoch as a secondary service.

The obvious question to ask next is "how well can velocities gridded from CORS perform?" As mentioned, intra-frame velocities will not be provided to users attempting to perform least squares adjustments of data ranging across large spans of time, but rather to provide a secondary service and for that reason, the question of how well they perform can be considered under the application of "moving a map from one epoch to another". This allows some flexibility in accuracy restrictions.

In such a case, examining the most egregious locations of intra-frame motion should help. Jarir Saleh (personal communication) gridded CORS linear velocities and compared them against the CORS themselves and found that, aside from occasional outliers (which need to be checked and possibly removed if the CORS data are erroneous), a grid of CORS velocities yields small *residual* intra-frame velocities. See Figure 11 for a horizontal example in the western USA:

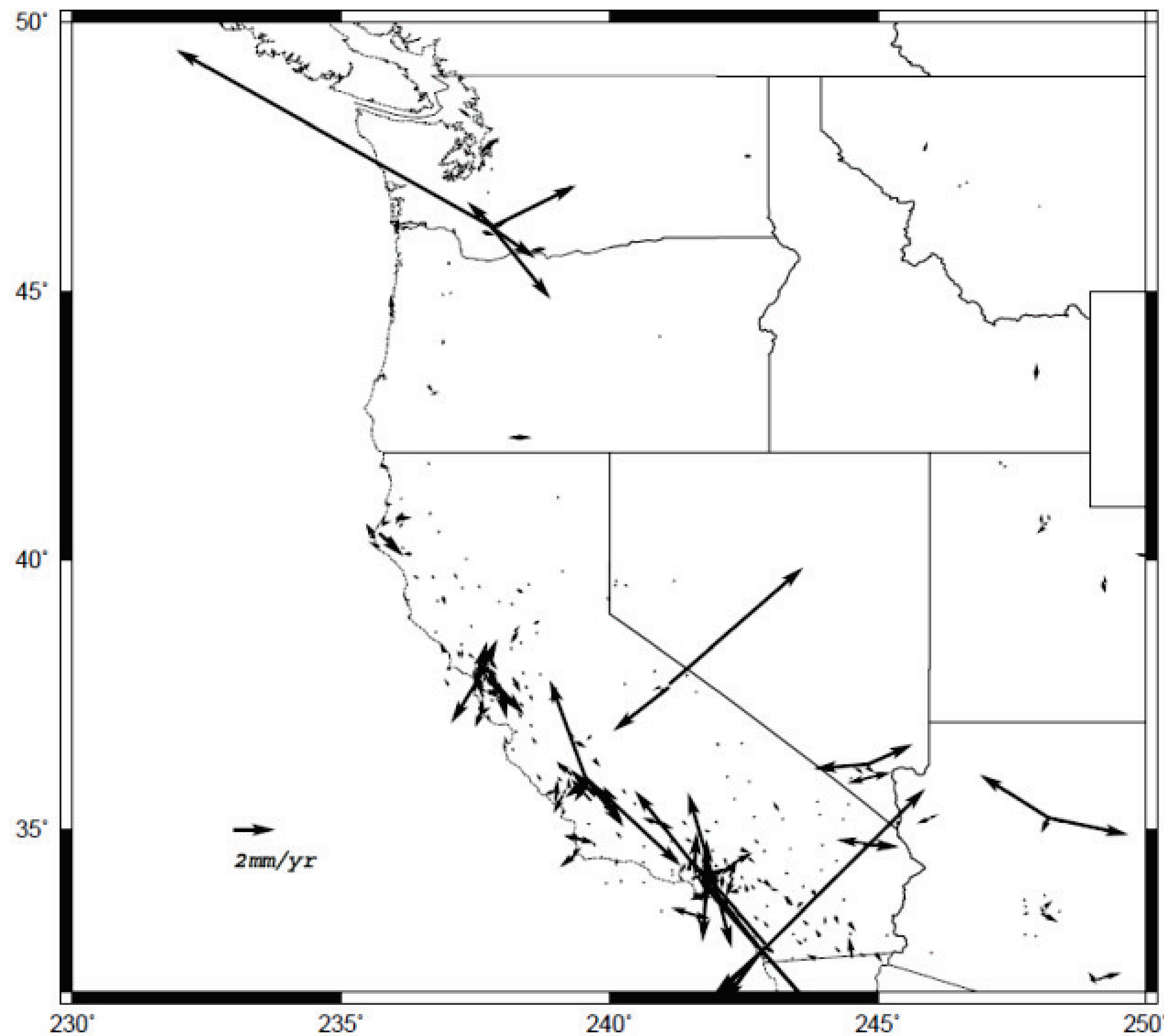

Figure 11: ***Residual*** **intra-frame horizontal velocities (tectonic plate rotation removed, followed by a removal of gridded intra-frame CORS-based horizontal velocities)**

What Figure 11 exemplifies is that, for the purposes of providing temporal transformations for maps and other geospatial products with accuracies looser than geodetic quality, a simple grid of CORS intra-frame velocities provides residuals that are small enough not to exceed 1-2 cm over about a decade. And if such CORS-based intra-frame velocity grids are updated on a 5-10 year interval, then these residuals will not necessarily grow to a level that has any significant impact on users work. That said, Figure 11 also shows several points (affected by earthquakes, deformation zones, rotating blocks along the plate boundary, etc.) that can exceed 2 mm/year for their residual intra-frame velocities. Such problem points will always prove difficult to model.

Mathematically speaking, these CORS based intra-frame velocities represent the [dx, dy, dz] vector of equation 13.

Therefore, as a service to the public, NGS will allow for the following steps for each of the four new terrestrial reference frames:

1) Cartesian coordinates (X_1, Y_1, Z_1) of a point occupied with a GNSS receiver will be computed using OPUS in an ideal frame (ITRF or IGS) at the epoch of the survey:

$$\begin{Bmatrix} (X_1, Y_1, Z_1)_t \ at \ CORS \\ GNSS \ reciever \ file \ from \ user \end{Bmatrix} \xrightarrow[OPUS \ processing]{} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \qquad (59)$$

2) Using the GRS-80 ellipsoid, these coordinates will be transformed into geodetic coordinates at the epoch of the survey:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t \xrightarrow[GRS-80]{} \begin{bmatrix} \phi_1 \\ \lambda_1 \\ h_1 \end{bmatrix}_t \qquad (60)$$

3) All four Euler Pole rotations will be applied to the Cartesian coordinates (see equation 19), yielding four sets of Cartesian coordinates, one for each terrestrial reference frame, at survey epoch. (While the software could be forced to try to "pick" the right frame, such choices seem best left to the user. At best the code might suggest which plate the user is on):

$$M_{NATRF2022}^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = \begin{bmatrix} x_{NATRF2022} \\ y_{NATRF2022} \\ z_{NATRF2022} \end{bmatrix}_t \qquad (61)$$

$$M_{CTRF2022}^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = \begin{bmatrix} x_{CTRF2022} \\ y_{CTRF2022} \\ z_{CTRF2022} \end{bmatrix}_t \qquad (62)$$

$$M_{PTRF2022}^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = \begin{bmatrix} x_{PTRF2022} \\ y_{PTRF2022} \\ z_{PTRF2022} \end{bmatrix}_t \qquad (63)$$

$$M_{MTRF2022}^{-1} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}_t = \begin{bmatrix} x_{MTRF2022} \\ y_{MTRF2022} \\ z_{MTRF2022} \end{bmatrix}_t \qquad (64)$$

4) Using the GRS-80 ellipsoid, these four sets of coordinates will be transformed into geodetic coordinates at the epoch of the survey:

$$\begin{bmatrix} x_{NATRF2022} \\ y_{NATRF2022} \\ z_{NATRF2022} \end{bmatrix}_t \xrightarrow[GRS-80]{} \begin{bmatrix} \phi_{NATRF2022} \\ \lambda_{NATRF2022} \\ h_{NATRF2022} \end{bmatrix}_t \qquad (65)$$

$$\begin{bmatrix} x_{CTRF2022} \\ y_{CTRF2022} \\ z_{CTRF2022} \end{bmatrix}_t \xrightarrow[GRS-80]{} \begin{bmatrix} \phi_{CTRF2022} \\ \lambda_{CTRF2022} \\ h_{CTRF2022} \end{bmatrix}_t \quad (66)$$

$$\begin{bmatrix} x_{PTRF2022} \\ y_{PTRF2022} \\ z_{PTRF2022} \end{bmatrix}_t \xrightarrow[GRS-80]{} \begin{bmatrix} \phi_{PTRF2022} \\ \lambda_{PTRF2022} \\ h_{PTRF2022} \end{bmatrix}_t \quad (67)$$

$$\begin{bmatrix} x_{MTRF2022} \\ y_{MTRF2022} \\ z_{MTRF2022} \end{bmatrix}_t \xrightarrow[GRS-80]{} \begin{bmatrix} \phi_{MTRF2022} \\ \lambda_{MTRF2022} \\ h_{MTRF2022} \end{bmatrix}_t \quad (68)$$

At this point, NGS has produced what it considers "geodetic quality" coordinates. The next set of coordinates, while provided as a service, should not be used as geodetic control for anyone actually performing geodetic control surveys.

5) Using the CORS-based intra-frame velocity model, these geodetic latitude, longitude and ellipsoid height coordinates will be moved backwards in time to the most recent "epoch of convenience", designated "t_c" for now. Such an epoch of convenience is likely to occur every 5 to 10 years. *Also note that there is no relation between any of these "epochs of convenience" and the value "t_0" at which the ideal frame and the four terrestrial frames are identical.*

$$\begin{bmatrix} \phi_{NATRF2022} \\ \lambda_{NATRF2022} \\ h_{NATRF2022} \end{bmatrix}_{t_c} = \begin{bmatrix} \phi_{NATRF2022} \\ \lambda_{NATRF2022} \\ h_{NATRF2022} \end{bmatrix}_t - \begin{bmatrix} d\phi_{NATRF2022} \\ d\lambda_{NATRF2022} \\ dh_{NATRF2022} \end{bmatrix}_{t,t_c} \quad (69)$$

$$\begin{bmatrix} \phi_{CTRF2022} \\ \lambda_{CTRF2022} \\ h_{CTRF2022} \end{bmatrix}_{t_c} = \begin{bmatrix} \phi_{CTRF2022} \\ \lambda_{CTRF2022} \\ h_{CTRF2022} \end{bmatrix}_t - \begin{bmatrix} d\phi_{CTRF2022} \\ d\lambda_{CTRF2022} \\ dh_{CTRF2022} \end{bmatrix}_{t,t_c} \quad (70)$$

$$\begin{bmatrix} \phi_{PTRF2022} \\ \lambda_{PTRF2022} \\ h_{PTRF2022} \end{bmatrix}_{t_c} = \begin{bmatrix} \phi_{PTRF2022} \\ \lambda_{PTRF2022} \\ h_{PTRF2022} \end{bmatrix}_t - \begin{bmatrix} d\phi_{PTRF2022} \\ d\lambda_{PTRF2022} \\ dh_{PTRF2022} \end{bmatrix}_{t,t_c} \quad (71)$$

$$\begin{bmatrix} \phi_{MTRF2022} \\ \lambda_{MTRF2022} \\ h_{MTRF2022} \end{bmatrix}_{t_c} = \begin{bmatrix} \phi_{MTRF2022} \\ \lambda_{MTRF2022} \\ h_{MTRF2022} \end{bmatrix}_t - \begin{bmatrix} d\phi_{MTRF2022} \\ d\lambda_{MTRF2022} \\ dh_{MTRF2022} \end{bmatrix}_{t,t_c} \quad (72)$$

In the above equation, the $d\phi$, $d\lambda$ and dh values come from interpolation from a CORS-based grid of intra-frame velocities.

It cannot be stressed strongly enough that the values provided in equations 69-72 should not be used as geodetic control by anyone performing geodetic surveys. They will, however, be very valuable for creating datum transformation tools such as NADCON.

9 Summary

Four new terrestrial reference frames, each one mathematically defined so that its latitude/longitude grid is rigidly rotating about an Euler pole for a specific tectonic plate, will be defined relative to some future IGS frame prior to 2022. These frames, being rigid and laid over a non-rigid crust mean that any velocities measured at geodetic control points which differ from plate rotation will be provided as residual intra-frame velocities on those points. Geodetic control in 2022 will be time-dependent, and coordinates can get "stale". In order to provide some (non-geodetic) information about these movements, NGS will provide a data-driven intra-frame velocity model, updated every 5-10 years (at "epochs of convenience") which will allow users to compare surveys and maps at different epochs, but only at non-geodetic accuracies.

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