

Technical Memorandum

WSR-88D Polar-to-HRAP Mapping

Richard A. Fulton

Hydrologic Research Laboratory
Office of Hydrology
National Weather Service
Silver Spring, Maryland

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Abstract

The WSR-88D rainfall algorithm generates a one-hour rainfall product that has been remapped from a local, radar-centered, polar grid into the national, quasi-rectangular HRAP grid of nominal grid size of 4 km x 4 km. This report describes the way that the polar-to-HRAP coordinate transformation is performed within the WSR-88D algorithm. It also compares the latitude-longitude grid cell locations based on the NEXRAD polar-to-HRAP transformation equations with the corresponding locations determined by the equations used in the River Forecast System and Stage III Precipitation Processing operational software applications that use these HRAP rainfall products for follow-on hydrologic processing within the National Weather Service. An error assessment is performed. These locations are also compared with those computed within the GENHRAP program developed by Reed and Maidment at the University of Texas. This information will serve as guidance for NWS users as well as users from other commercial or government organizations who wish to use high resolution WSR-88D HRAP rainfall estimates within GIS-based distributed hydrologic models or other applications.

Results indicate that the WSR-88D HRAP rainfall product contains areally averaged rainfall over the HRAP grid box where a grid box is defined as the area enclosed by four contiguous HRAP grid points (I, J) where I, J are integers. Thus the HRAP rainfall estimates are actually centered at HRAP grid points (I+0.5, J+0.5). All rainfall estimates on the polar grid whose cell centers lie within the boundary of an HRAP grid box are averaged to become the rainfall estimates for that HRAP box regardless of how much of the polar grid box may lie outside of the given HRAP grid box. This is a simple and efficient method of remapping the polar data into the quasi-rectangular HRAP grid within the operational rainfall algorithm.

It is also found that the geo-spatial registration of the HRAP grid mesh relative to the earth's surface based on the NEXRAD equations is consistent with, though not in exact agreement, with the positioning based on the equations used in Stage III Precipitation Processing and the NWS River Forecast System. Based on testing for three WSR-88D radars widely separated across the U.S., the mean spatial displacement between the two HRAP grids increases with range but averages about 0.3 km ($< 1/10^{\text{th}}$ HRAP grid box) averaged over all grid points that fall within 230 km of a given radar. The University of Texas' GENHRAP program that translates HRAP grid locations to latitude-longitude locations produces an HRAP grid whose latitude-longitude coordinates are exactly registered with those of the HRAP grid produced by Stage III Precipitation Processing and NWS River Forecast System software.

I. Introduction

The National Weather Service (NWS) has completed the installation of about 160 WSR-88D radars across the U.S. as a part of the modernization program called NEXRAD (Next Generation Radar). Resident on these WSR-88D computers is a rainfall algorithm called the Precipitation Processing System (PPS) (Fulton et al. 1998) that computes rainfall estimates from reflectivity measurements collected on a polar grid centered on the radar. This grid has a fixed spatial resolution of 2.0 km in range by 1.0 degree in azimuth. The rainfall estimates computed in the PPS represent spatial averages over these grid cells whose sizes depend on the distance from the radar. A 2-km by 1-deg grid cell ranges in size from 2 by 4 km at the farthest range of the radar (230 km) to 2 by 2 km at mid ranges (115 km) to 2 km by 0.3 km at close ranges (50 km).

One of the products of this algorithm is an Hourly Digital Precipitation Array (called HDP or DPA) product which is a digital one-hour rainfall accumulation product that has been remapped from the radar-centered polar grid onto a polar stereographic projection. This projection is called the HRAP (Hydrologic Rainfall Analysis Project) projection and is a quasi-rectangular grid whose cell size is nominally 4 km on a side but ranges from about 3.5 km in southern contiguous U.S. latitudes to about 4.5 km in northern contiguous U.S. latitudes (Greene and Hudlow, 1982; Schaake 1989; NWSRFS, 1998). This HRAP grid covers the 48 conterminous states. Appendix A contains selected pages from NWSRFS (1998) describing this grid. Reed and Maidment (1998) describe mapping procedures to register the HRAP grid over the earth's surface.

The HRAP grid is utilized by the NWS so that the polar rainfall estimates from the WSR-88D radars can be mosaicked onto a common grid across the U.S. for follow-on hydrologic applications to support their river modeling activities and flood forecasting mission. The NWS Hydrologic Research Laboratory has developed custom software called "Stage III Precipitation Processing" which performs this mosaicking of multiple HDPs (plus other calibration and quality control procedures utilizing rain gages) over sub-regions of the U.S. served by the NWS River Forecast Centers (RFC) (Shedd and Fulton 1993; Fulton et al 1998). Stage III is executed at the RFCs on a Unix workstation external to the WSR-88D computer, and the resulting mosaicked radar/gage rainfall estimates on the HRAP grid are then input into the

NWS River Forecast System (NWSRFS), a suite of operational numerical hydrologic models used to forecast stream stage.

The two main objectives of this study are as follows:

1) describe how polar rainfall estimates are mapped to the HRAP grid within the PPS FORTRAN code, and

2) compare how the PPS code defines the HRAP grid with the way that Stage III / NWSRFS and the University of Texas' GENHRAP program define it. This is to verify that the HRAP equations in both the WSR-88D and external computers are consistent in their registration of the HRAP grid mesh relative to the earth's latitude-longitude grid. The grids are displayed and compared using ARC/INFO Geographic Information Systems.

II. NEXRAD Technical Specifications for Mapping

There are two official NEXRAD documents that describe in words and/or equations the specifications for polar-to-HRAP mapping. The first is the NEXRAD Algorithm Report which defines the “algorithm enunciation language” (AEL) for all the WSR-88D algorithms, including the PPS. This document was written by the NWS in the early to mid-1980’s and was provided to the NEXRAD contractor (Unisys Corp.) who built the radar and wrote all the software. The AEL describes in paragraph and pseudo-code format the way that the mapping is to be performed. Included in Appendix B is a copy of the selected pages from the PPS Products algorithm contained in this document describing polar-to-HRAP mapping.

The second NEXRAD document containing specifications for the mapping is called the “B-5 Specification”. This document written by the NEXRAD contractor contains a much higher level of technical detail than the Algorithm Report, including all of the transformation equations. This document was used to write the FORTRAN code. Appendix C is a copy of the relevant pages from this document.

The HRAP grid is a nested grid within the NWS’s LFM (Limited Fine Mesh) grid. The LFM is the name of an early atmospheric numerical model used by the NWS, and the grid it used was called the LFM grid. A finer mesh version of the LFM grid (with grid cell length exactly $1/40^{\text{th}}$ of the LFM grid) was adopted by the PPS, and this is what is called the HRAP grid. Both of the NEXRAD documents above use exclusively the term “ $1/40^{\text{th}}$ LFM” grid to describe the HRAP grid, but they are one and the same.

According to the Algorithm Report (see Appendix B),

“The hourly running totals or clock hour totals on the $1/40^{\text{th}}$ LFM grid are obtained by determining the mean, after conversion into absolute units (mm), of all adjusted ACCUMULATION (Hourly) sample volumes whose polar coordinate centers fall within each $1/40^{\text{th}}$ LFM grid box.”

and

“The Limited Fine Mesh (LFM) grid is a rectangular grid commonly used by the National Weather Service which is based on a polar stereographic projection. An LFM grid box represents an area whose size and shape varies with latitude. Therefore the size and shape of the grid boxes will vary slightly over the area covered by the radar and even more from radar to radar (3.5 to 4.5 km over the conterminous U.S. for the 1/40th LFM grid. The 1/40th LFM grid boxes used here are defined to have 1/40th LFM grid points as their centers and a mesh length of 4.7625 km at 60 degrees N latitude....).”

The important points to note here are that, according to these specifications, 1) the polar rainfall estimates for all polar grid cell centers that fall within an HRAP grid cell are averaged for that HRAP grid cell, and 2) those averaged rainfall estimates are centered over an HRAP grid *point* (I, J) (where I, J are integers) as opposed to a grid *box* (defined by 4 contiguous grid points) whose center is at (I+0.5, J+0.5). *However, it turns out after close examination of the actual FORTRAN code that the PPS code does not actually implement specification #2 in the manner described as will be discussed below.*

III. PPS Implementation of the NEXRAD Specifications

In order to verify that the actual PPS FORTRAN code implements the AEL specifications as defined above, the code was examined in detail. Two of the most relevant NEXRAD subroutines are:

A31341__BUILD_LFM_LOOKUP : determines which HRAP grid box each range-azimuth rainfall grid cell falls within for a given radar.

A31467__LFM_CART : averages all polar rainfall estimates that fall within an HRAP grid cell using a lookup table defined previously in A31341.

A. A31341__BUILD_LFM_LOOKUP

The subroutine A31341__BUILD_LFM_LOOKUP is executed just once within the Rate-Accumulation task of the PPS when the WSR-88D Radar Product Generator (RPG) is started (either warm or cold startup). It builds a FORTRAN integer common block array (“lookup table”) of size 360 by 115 (360 azimuth bins of 1 degree width and 115 range bins of 2 km depth out to 230 km maximum range) that maps each and every polar grid cell into one and only one HRAP grid cell using the radar’s latitude and longitude and the B-5 Specification equations defining the HRAP grid (see Appendix C). This involves looping over azimuths 0.5, 1.5, 2.5, ..., 358.5, 359.5 degrees and ranges 1.0, 3.0, 5.0, ..., 227.0, 229.0 km which represent the centers of the 1-deg by 2-km polar grid cells. The transformation equations convert these range-azimuth grid point locations to HRAP coordinates (real numbers), and then by truncating these real numbers to integers, the upper-left corner of the HRAP grid box for which that range-azimuth bin is assigned is thereby determined. They are then saved off in a common block array named LFM40GRID for later use in A31467

(see below). Truncating locates the upper-left corner of the HRAP box because the NEXRAD software arbitrarily defines the I-axis as positive to the right (east) and the J-axis as positive downward (south).

The major conclusion to be made here based on examination of the actual NEXRAD software is that the HRAP rainfall estimates represent areally averaged rainfall in which the center of that area is located at HRAP grid point (I+0.5, J+0.5), i.e., the center of an HRAP grid box, not one of its corners. This disagrees with the AEL specification #2 above.

The HRAP grid can be envisioned as a fixed rectangular mesh overlaid on the earth's surface in which locations lying on the mesh intersections are associated with integer-multiple coordinates (I, J) while point locations off the mesh intersections are associated with real number coordinates (X,Y). The HRAP mesh is arbitrarily positioned by the NEXRAD specifications such that the North Pole falls on the grid point (4330, 4330). Note that Appendix C states that the North Pole falls at point (433, 433), but this is referring to the location on the ¼ LFM grid, so a scale factor of 10 must be applied to these coordinates to convert to them to the 1/40th LFM (HRAP) grid.

For each individual radar a smaller square subset of the national HRAP grid mesh is extracted that has a fixed size of 131 by 131 grid cells which is large enough to encompass the circular 230 km maximum range scanning domain of any WSR-88D radar. This local grid array is selected from the national HRAP grid array such that the radar falls somewhere within the center grid box (66, 66). The location of the radar may fall anywhere within this center grid box depending on where that radar happens to fall relative to the fixed HRAP grid mesh. This is a key assumption of both the NEXRAD "encoding" software for the HDP product and the reverse "decoding" software on external computers. As long as these two computer algorithms use identical HRAP grid mapping equations and identical radar latitude-longitude locations, the local 131 x 131 HDP rainfall array should be positioned correctly relative to the earth in the external applications.

Within the WSR-88D computer, the square 131 x 131 array of rainfall estimates for that radar is created where each grid box is uniquely identified by an (M, N) coordinate where M and N are integers and the indexing begins in the upper left (northwest) corner and ends in the lower right (southeast) corner. That is, the upper-left-most HRAP grid box is defined as (1,1) while the lower-right-most grid box is defined as (131,131). Note that in this case these integer coordinates reference the area-averaged rainfall for a grid *cell*, not a grid *point*. For programming convenience, the actual FORTRAN code defines a one-dimensional array of length 17,161 (= 131x131). Typically only about 9500 of these 17,161 array locations will fall within the maximum 230 km range where rainfall estimates are computed.

The polar-to-HRAP lookup table that maps range-azimuth to HRAP coordinates does not change over time for a given radar because the polar grid in which PPS rainfall estimates are computed is fixed at 2-km by 1-degree, thus this subroutine needs only to be executed once when the computer is started. The HRAP grid cell that a particular range/azimuth cell is mapped into will vary from radar to radar because both the HRAP grid cell size and orientation vary with latitude and

longitude, and therefore each radar has its own unique lookup table to map from range-azimuth to HRAP grid location.

Frequently a number of different polar grid cells will map into a single HRAP cell, especially at short ranges because of the higher spatial resolution of the rainfall estimates close to the radar. At the farthest ranges, usually two and possible only one polar grid cell will be mapped into a given HRAP cell. The HRAP grid size corresponds approximately, by design, to the largest dimension of the polar range and azimuth resolution of the PPS rainfall arrays at the farthest range of 230 km. Figure 1 shows the HRAP grid overlaid on top of the polar grid for the Denver, Colorado WSR-88D at two selected ranges, one close to the radar (Fig. 1a) and one at far ranges near 230 km (Fig. 1b).

Fig. 1. The NEXRAD HRAP grid (thick square mesh) for the Denver, CO (FTG) WSR-88D radar overlaid on top of the polar 1-deg by 2-km PPS rainfall grid for locations a) near the radar (upper left), and b) near the maximum 230 km radar range to the southeast.

B. A31467__LFM_CART

Once a polar-to-HRAP lookup table is created and available after radar startup, the subroutine A31467__LFM_CART is executed within the PPS Products algorithm task every volume scan using as input the polar one-hour rainfall accumulation array ending at that volume scan and the common block array LFM40GRID defined above. It is within this subroutine where all polar rainfall estimates assigned to each HRAP grid box are averaged. The output is a 131 x 131 array of one-hour rainfall estimates on the HRAP grid centered on the radar.

IV. Comparing the registration of the NEXRAD and Stage III HRAP grid meshes

We have just shown in the previous section that the NEXRAD software averages polar rainfall estimates over areas defined by the HRAP boxes, not centered on HRAP grid points. In order that these remapped rainfall estimates be properly positioned relative to the earth in applications external to the WSR-88D, it is critical that both the NEXRAD software and external software define the HRAP grid mesh in exactly the same way such that there are no relative displacements in the mesh on the earth's surface. That is, it is important that the mapping transformation procedures on both the WSR-88D and external computers be "forward-backward" compatible so that the rainfall is ultimately mapped into the identical position on the earth's surface to where the original measurements were collected. This section examines this compatibility issue.

Stage III and NWSRFS use a FORTRAN subroutine called "SBLGDD" to transform back and forth between HRAP coordinates and latitude-longitude. The NEXRAD software in subroutine A31341__BUILD_LFM_LOOKUP as defined in the

B-5 Specification does a one-way "forward" conversion from radar range-azimuth of the polar grid cell centers to the corresponding HRAP coordinates (real numbers).

The goal in this section is to do the reverse transformation from integer-multiple HRAP coordinates (I, J) (i.e., the grid points representing the mesh intersections) to range-azimuth using the NEXRAD equations and then ultimately latitude-longitude (see conceptual diagram below). Since the NEXRAD software does not do the last step of converting range-azimuth to latitude-longitude, an additional computational step is necessary using an additional computer program to do this last conversion step. Then the resulting NEXRAD-based latitude-longitude locations of the HRAP grid points can be compared with the same information

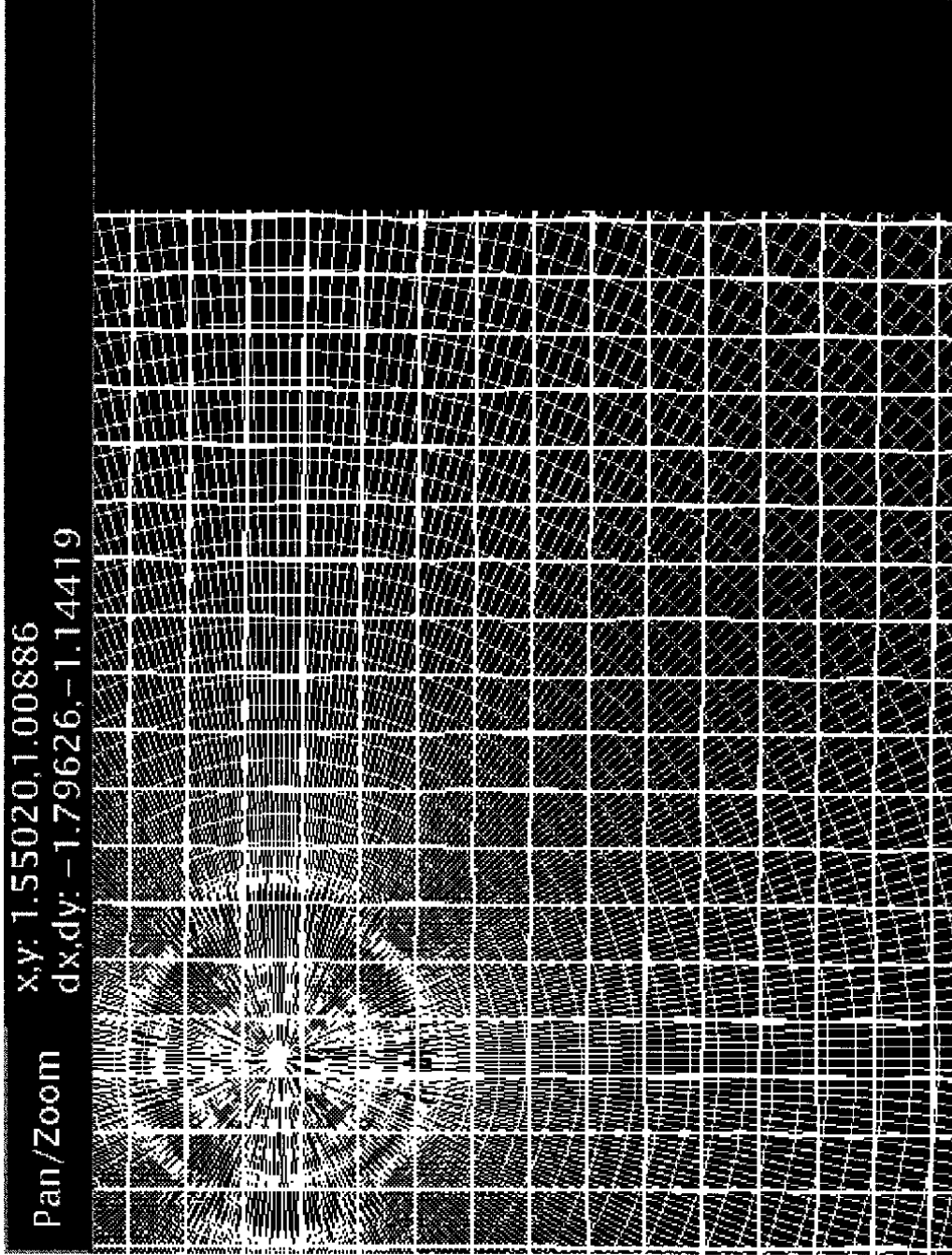


Figure 1a

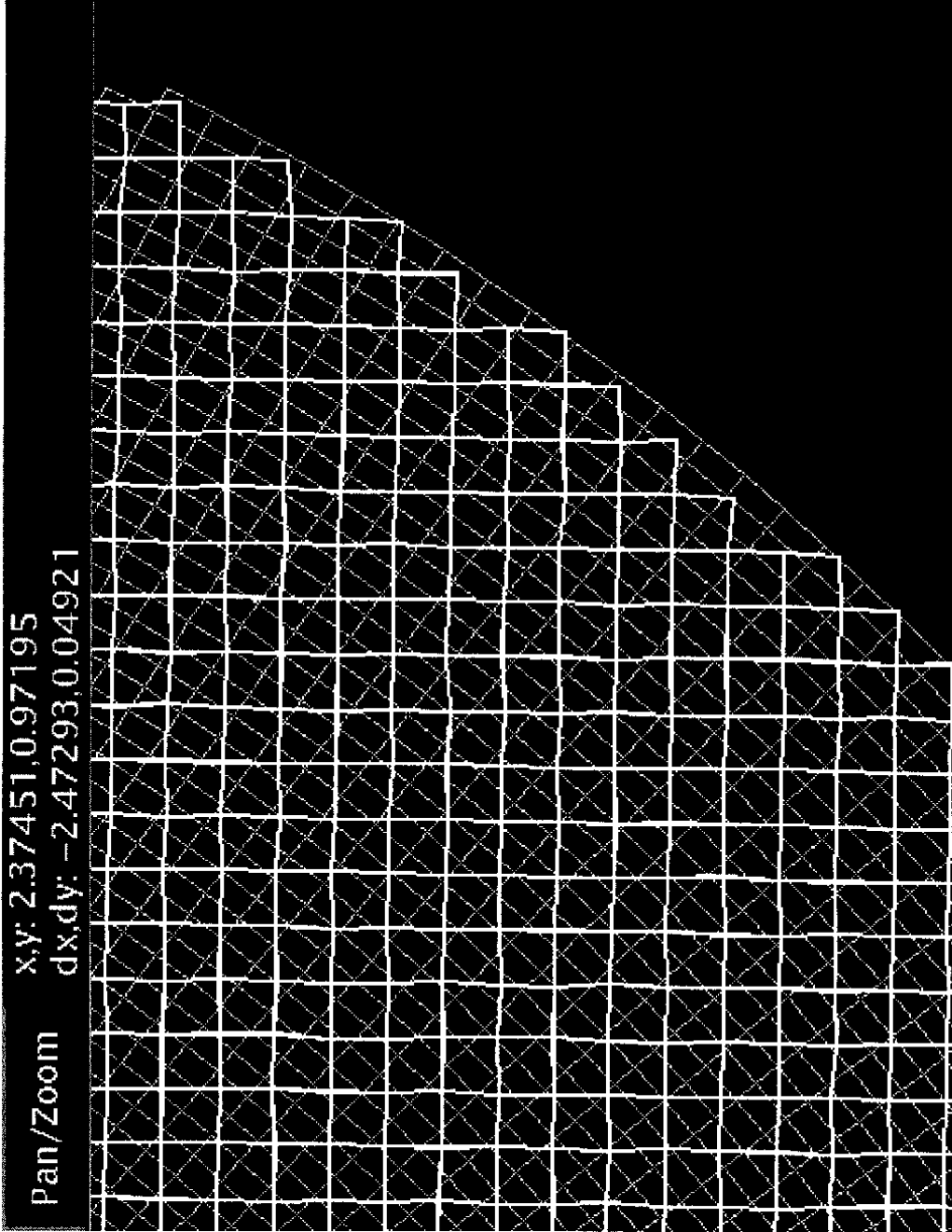
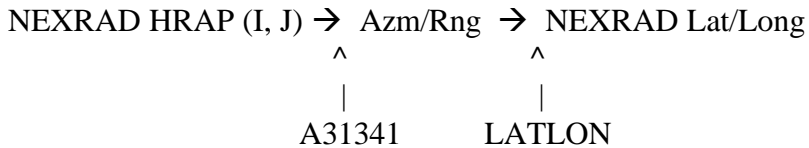


Figure 1b

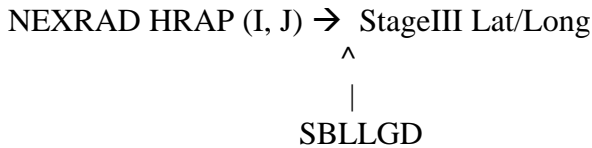
derived from the SBLLGD program using the exact same HRAP grid points as input to both procedures. If the two programs produce the same latitude-longitude pairs, then they are compatible. If not, then there exists some undesirable incompatibility that will result in inherent geo-referencing errors when the NEXRAD rainfall estimates are mapped onto the earth's surface in Stage III or NWSRFS using the SBLLGD subroutine. The latter has been found to be the case though the errors are not severe as will be discussed shortly.

Summary of the analysis procedures to be discussed next:

Step 1: Compute the range-azimuths of NEXRAD-based HRAP grid points from the NEXRAD equations using subroutine A31341 and then transform them to latitude-longitudes using LATLON

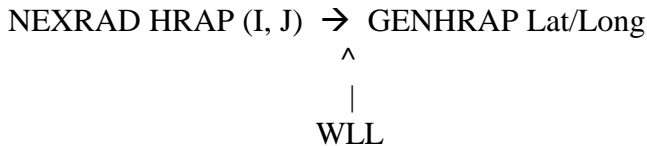


Step 2: Compute the latitude-longitudes of NEXRAD-based HRAP grid points using the SBLLGD subroutine



Step 3: Compare NEXRAD Lat/Long and StageIII Lat/Long to determine compatibility

Step 4: Compute the latitude-longitudes of NEXRAD-based HRAP grid points using the WLL subroutine of the GENHRAP program and compare with NEXRAD and Stage III Lat/Long



Description of Step 1:

The original goal in Step 1 was to take the existing “forward” equations from the NEXRAD B-5 Specification (i.e., those that convert from range-azimuth to HRAP coordinates) and derive the “reverse” equations (i.e., the equations to convert from HRAP coordinates back to range-azimuth). After examining the equations, this appeared to be a non-trivial exercise. Therefore it was decided to use a simpler, though less exact, iterative procedure that used the existing forward equations from NEXRAD to get *estimates* of the range and azimuth of the integer-multiple HRAP grid coordinates (I, J). Then a conversion from estimated range-azimuth to estimated latitude-longitude could be done.

Several FORTRAN programs designed to be run in sequence were written to do the processing in Steps 1-3 (AZRAN2HRAP, AZRANCOR, and GETLALOHRAPCOR). AZRAN2HRAP calls a slightly modified version of the NEXRAD subroutine A31341__BUILD_LFM_LOOKUP (hereafter referred to simply as “A31341”). The equations within A31341 to convert range-azimuth to HRAP coordinates were embedded within a FORTRAN loop over range and azimuth at a resolution ten times greater than the resolution within the existing subroutine, i.e., range was incremented every 0.2 km from 0 to 230 km, and azimuth was incremented every 0.1 degrees from 0 to 360 degrees. This program produced a list of HRAP (X,Y) coordinates (real numbers) for each range-azimuth pair on this high resolution polar grid.

After sorting this list using the Unix SORT command, another program, AZRANCOR, finds the single range-azimuth pair that is closest to each integer-multiple HRAP coordinate (I, J) for all HRAP grid points within 230 km range of the radar. The resulting HRAP coordinates, called (I', J'), will be no more than 0.1 km in range and 0.05 degrees in azimuth (i.e., half of the chosen resolution given above) from the corresponding integer (I, J) coordinates. 0.05 degrees azimuth equals a distance of 0.20 km at the maximum range of 230 km and even less than that at closer ranges. Thus the maximum spatial displacement error between the estimated HRAP coordinates (I', J') and the exact coordinates (I, J) is about 0.22 km ($=\sqrt{0.20^2 + 0.10^2}$) at 230 km range, about 0.14 km at mid ranges, and even less closer to the radar. Such an error of one- or two-tenths of a kilometer was deemed accurate enough for this analysis. If lower errors were desired, then the 0.2 km range and 0.1 degree azimuth increments could be easily decreased at the expense of increased computer processing time.

At this point, we now have range-azimuths of all the HRAP grid points (or at least very close approximations to them) and have in essence “reversed” the NEXRAD equations. Since we ultimately want latitude-longitudes of these grid points for comparison, and since the NEXRAD equations in A31341 do not compute that information, another subroutine is necessary to translate range-azimuth to latitude-longitude. A FORTRAN subroutine called LATLON, obtained from NASA Goddard Space Flight Center, is called within GETLALOHRAPCOR to perform this translation using the computed range-azimuth pairs from A31341. This subroutine does transformations back and forth between radar range-azimuth-elevation and latitude-longitude, taking into account the earth’s curvature (assuming the earth is an oblate

spheroid) and (optionally) refraction of the radar beam through the atmosphere using the “4/3rds earth radius” model. For this study, the beam elevation angle was set to 0.5 degrees, and the option to account for refraction was toggled both on and off in separate sensitivity runs.

The net result of this multi-program processing is a list of the following items valid for each HRAP grid point (I, J) within 230 km of a given radar:

- 1) NEXRAD-based HRAP coordinates,
- 2) range-azimuth coordinates, and
- 3) NEXRAD-based latitude-longitude.

Recall that these locations are very near (within a few tenths of a kilometer or less) but not exactly coincident with the HRAP grid points (I, J) due to the use of the iterative, non-exact, “forward” transformation procedure.

Three radars were used for testing purposes: Denver, Colorado (FTG), Caribou, Maine (CBW), and San Diego, California (NKX) WSR-88Ds.

These latitude-longitude grid points were then combined into a series of four-sided polygons representing the HRAP grid mesh and then input into ARC/INFO GIS using the GENERATE command. The resulting polygon coverage was projected from the geographic (latitude-longitude) coordinates into the Universal Transverse Mercator (UTM) coordinate system using a NAD 27 datum and Clarke 1866 spheroid.

We now have completed Step 1 and have produced a polygon coverage of the HRAP grid as computed by the NEXRAD equations defined in the NEXRAD B-5 Specification, and this coverage is one of our major products of this analysis to be used for upcoming comparisons.

Description of Step 2:

In this step, the latitude-longitudes of the NEXRAD-based HRAP coordinates (I, J) determined above were computed by using a slightly modified version of the SBLLGD subroutine from Stage III / NWSRFS so that the NEXRAD-derived HRAP mesh could be compared to the Stage III-derived HRAP mesh. This was accomplished by placing a call to SBLLGD within GETLALOHRAPCOR using as input the “approximate” HRAP coordinate (I, J) associated with the closest range-azimuth location to the integer-multiple HRAP grid point. This was done instead of inputting the “exact” (i.e., integer) HRAP (I, J) coordinates so that the errors caused by using the non-exact, iterative procedure would not be included in the comparison.

Because the NEXRAD and SBLLGD software have a different numbering (indexing) convention for the HRAP grid points, it was first necessary to translate the NEXRAD HRAP coordinates computed above to the SBLLGD-compatible coordinates before calling SBLLGD. The reason for this is two-fold: 1) NEXRAD equations assume the North Pole is at the HRAP location (4330, 4330), while the SBLLGD subroutine assumes it is at (401, 1601). 2) NEXRAD assumes HRAP coordinates increase to the east and south, while the SBLLGD subroutine assumes they increase to the east and north. A simple algebraic translation is performed to

resolve the disparate numbering conventions of the two programs before the NEXRAD HRAP coordinates are input into the SBLG sub-routine.

Just as was done with the NEXRAD-derived latitude-longitudes computed in Step 1, the SBLG-derived latitude-longitudes are combined into four-sided polygons and then ingested into ARC/INFO using the GENERATE command, and a polygon coverage is created in the same UTM coordinate system as was done previously. Figure 2 shows the two HRAP polygon coverages for the Denver radar. Rainfall attributes could be easily attached to the GIS coverages if desired within ARC/INFO.

Fig. 2. Comparison of the NEXRAD and Stage III HRAP grids for the Denver, CO radar for locations a) immediately surrounding the radar (the radar is in the center box), b) near the 230 km range to the southeast, and c) near the 230 km range to the northwest. The solid line corresponds to the NEXRAD grid, while the dashed line corresponds to the Stage III (SBLG) grid. The grids have both been projected into the identical UTM coordinate system from their original geographic (latitude-longitude) coordinate system for display purposes. The grid cell sizes are approximately 4 km.

Description of Step 3:

Two main results were found by comparing the two coverages created in Steps 1 and 2 for each of the three radars. First, it was determined that the NEXRAD and SBLG HRAP grids were aligned properly relative to each other, i.e., a given NEXRAD HRAP grid point coordinate (say e.g., (100, 200)) had *nearly* the same latitude-longitude as that same coordinate's latitude-longitude as computed using the SBLG routine. There were no significant offsets from one grid to the other such as, for example, one grid is offset relative to the other by one or more grid cell sizes in any direction.

Second, it was found that even though there are no major offsets, there are small spatial displacements averaging less than 1/10th of an HRAP grid cell size (i.e., less than 0.4 km) between the two grids (thus the statement above "nearly the same") as can be seen in Fig. 2. The mean, minimum, and maximum grid point-by-point differences in latitude and longitude between all HRAP grid points within 230 km radar range in the two grids were computed, and then these differences were translated to distance assuming a spherical earth of radius 6371 km (see Table 1).

Table 1. Statistics of the spatial displacement (in kilometers, regardless of the displacement direction) between NEXRAD and StageIII HRAP grid points lying within 230 km range of each of the San Diego, CA, Denver, CO and Caribou, ME WSR-88D radars.

<u>Radar</u>	<u>Mean absolute diff.</u>	<u>Max. absolute diff.</u>	<u>Min. absolute diff.</u>
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San Diego	0.20	0.31	0.00
Denver	0.26	0.40	0.00
Caribou	0.32	0.48	0.00

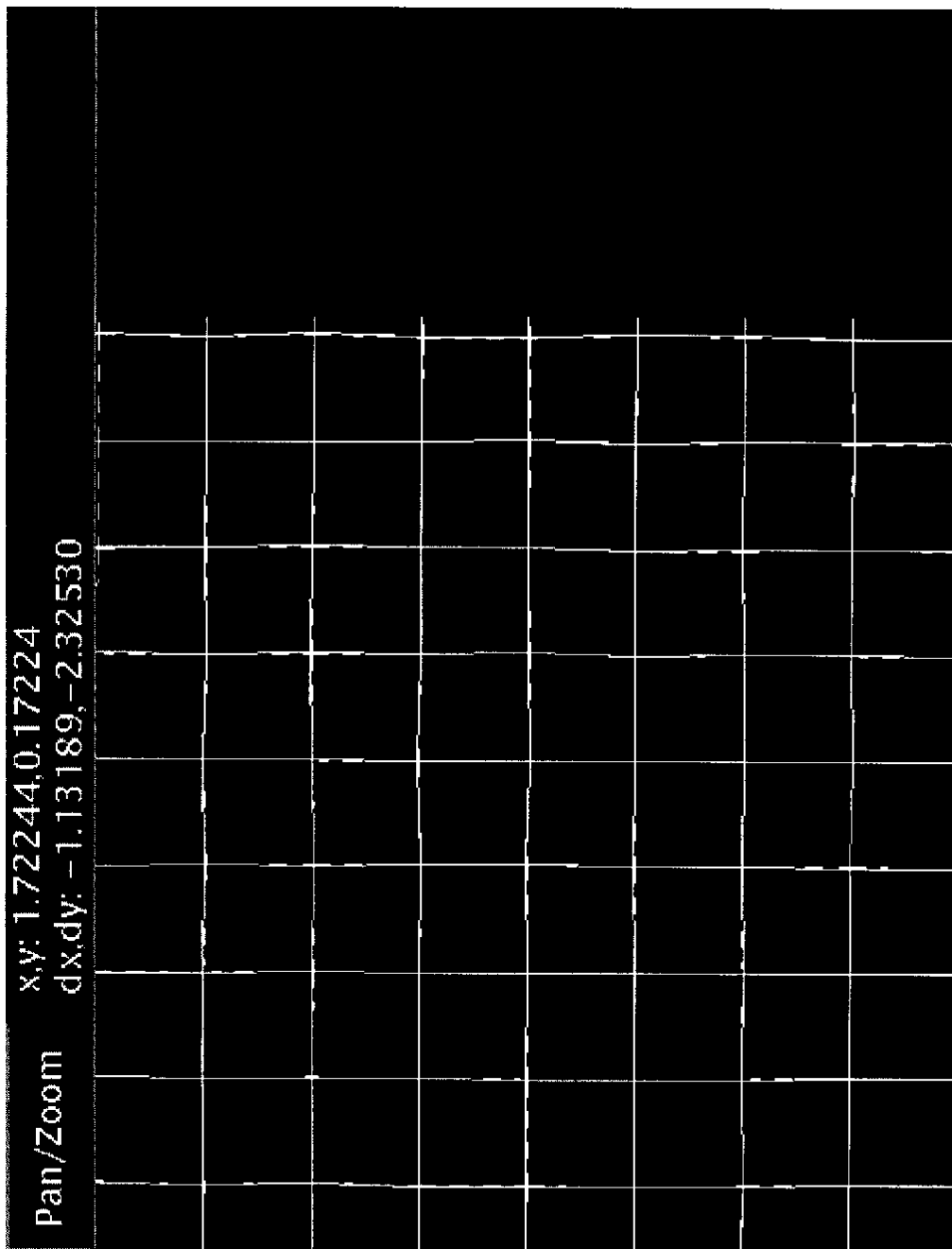


Figure 2a

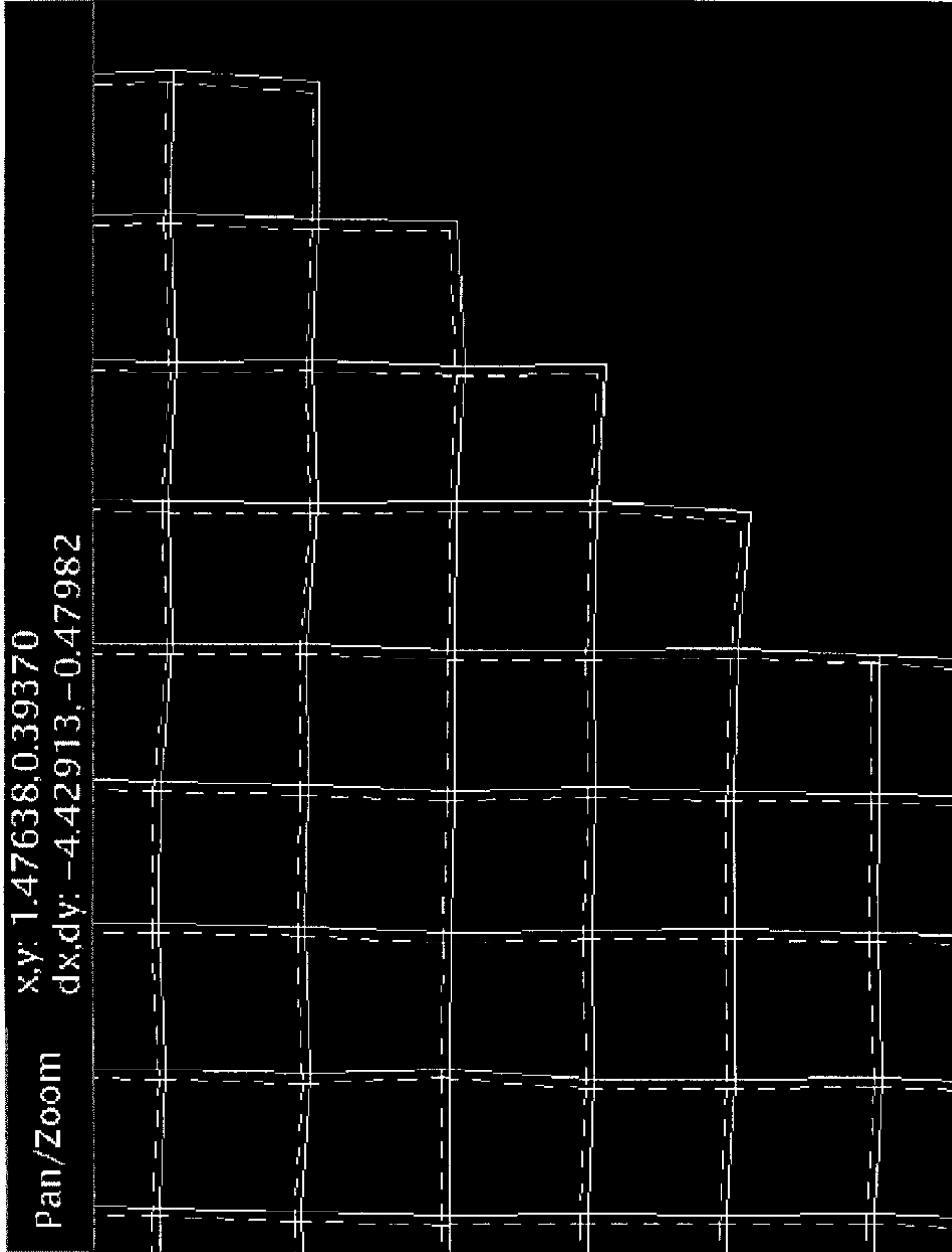


Figure 2b

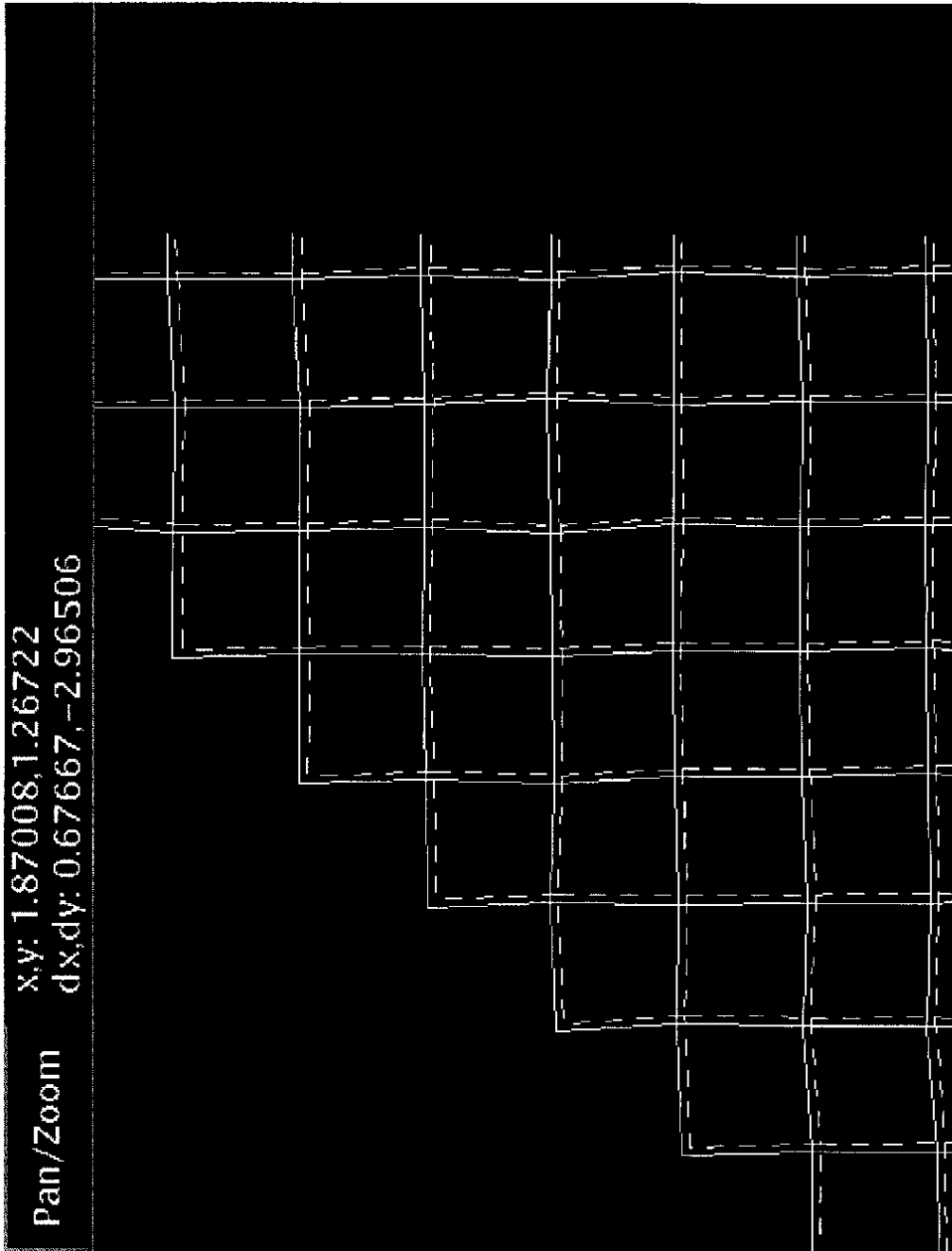


Figure 2c

The HRAP grid point locations (representing one corner of an HRAP grid box) as computed by the NEXRAD equations differ by about 0.3 km on average from the locations computed by Stage III / NWSRFS. This difference increases approximately linearly with range from a given radar as shown in the range profiles in Fig. 3.

Fig. 3. Profiles of the displacement errors (km) between the NEXRAD and Stage III HRAP grids for the Caribou (CBW), Denver (FTG) and San Diego (NKX) radars. These profiles are for the middle row (row 66) of the 131x131 HRAP array that passes through the radar from roughly west to east. The left and right edges correspond to ranges of roughly 220 km, and the radar is in the middle.

The difference also varies from one radar to another with a tendency to differ more as the radar's latitude location increases to the north. Fig. 2a shows that the two HRAP grids are closely aligned at near ranges, while at farther ranges (Figs. 2b and 2c) the difference increases. The Stage III grid points are always displaced *toward* the radar relative to the NEXRAD grid points for all azimuths around the radar except near the radar where they match well.

Note that these computed errors do *not* contain the earlier errors of several tenths of a kilometer because the "estimated" NEXRAD HRAP latitude-longitudes were used as input when computing the corresponding SBLGDLG latitude-longitudes. The errors shown in the table above are due solely to the differences in the mapping equations between the NEXRAD and SBLGDLG software. The reasons why there are differences have not been examined in this study.

There are also no systematic displacements in one particular direction between the grid points of the two HRAP grids. Table 2 shows the same information as Table 1 except that the two latitude and longitude components of the displacement are shown. The mean latitude and longitude differences were both very close to zero, and the minimum and maximum differences were evenly spaced on both sides of the mean. This is consistent with Fig. 2 and the observation that the Stage III grid boxes are shifted inward toward the radar for all radar azimuths. The maximum displacement in both latitude or longitude was about 0.4 km, or roughly 1/10th of an HRAP grid cell size.

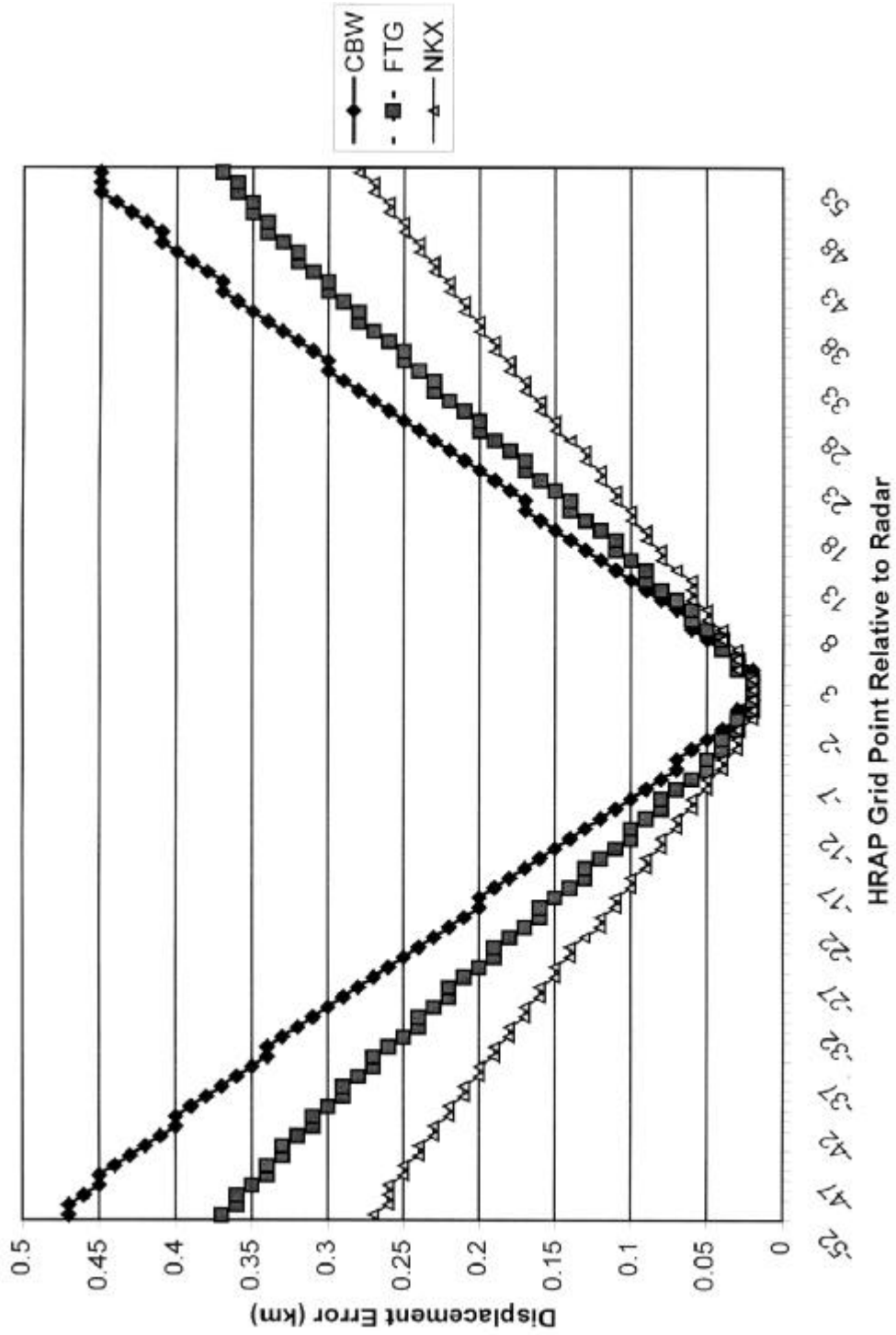


Figure 3

Table 2. Statistics of the spatial displacement in terms of latitude-longitude and kilometers between NEXRAD and StageIII HRAP grid points lying within 230 km range of each of the San Diego, Denver, and Caribou WSR-88D radars.

Radar	<u>Mean long. diff.</u>		<u>Min. long. diff.</u>		<u>Max. long. diff.</u>	
	(deg)	(km)	(deg)	(km)	(deg)	(km)
San Diego	2.7E-6	0.00	-0.00315	-0.29	0.00314	0.29
Denver	7.0E-7	0.00	-0.00446	-0.38	0.00448	0.38
Caribou	5.9E-6	0.00	-0.00604	-0.46	0.00605	0.47

Radar	<u>Mean lat. diff.</u>		<u>Min. lat. diff.</u>		<u>Max. lat. diff.</u>	
	(deg)	(km)	(deg)	(km)	(deg)	(km)
San Diego	1.2E-4	0.01	-0.00247	-0.27	0.00280	0.31
Denver	9.6E-5	0.01	-0.00328	-0.36	0.00355	0.39
Caribou	6.8E-5	0.01	-0.00405	-0.45	0.00433	0.48

Such displacement errors of several tenths of a kilometer (at most) can be considered negligible for most applications. This is especially true when you reconsider Fig. 1. Recall that the PPS maps a polar rainfall cell into an HRAP box based on the central location of that polar cell. Fig. 1 shows that such a relatively simplistic approach results in rainfall estimates that are actually not always centered exactly over the center of each HRAP grid box, especially at far ranges (Fig. 1b) where the NEXAD-StageIII mapping errors are highest anyway. Scanning over Fig. 1b, it is obvious that this mapping procedure may in fact produce rainfall that can be as much as *one or two kilometers* (half of an HRAP grid box) offset from the HRAP grid cell center for a given HRAP cell if the polar grid cell centers happen to fall very near the boundary of the HRAP grid box. This is only true at the farthest ranges of the radar. This makes the several-tenths of a kilometer errors discussed above seem trivial.

A much more elegant and exact approach (though much more computer intensive) would be to use the polygon overlay capabilities of GIS to partition the polar grid cells along the HRAP grid boundaries instead of lumping the whole polar cell into one HRAP bin. The rainfall that falls within a given polar grid cell that straddles an HRAP boundary can then be partitioned properly between two adjacent HRAP boxes. Such a capability is well beyond the capability of the existing WSR-88D because GIS software functionality is not resident on the RPG.

Description of Step 4:

As a last step, a comparison was done between the HRAP latitude-longitude locations as computed by the GENHRAP program created at the University of Texas (Reed and Maidment, 1998; Reed and Maidment, 1995). These authors have placed their GENHRAP program on a web site for free download, and the program has been run for the 230 km range domain of the Denver WSR-88D radar. In particular, the subroutine WLL within the GENHRAP program was extracted, slightly modified, and called within the GETLALOHRAPCOR program to compute latitude-longitude

coordinates from the NEXRAD HRAP coordinates (I, J) as was done above in Step 2. The results show that the computed latitude-longitudes match those computed by SBLLGD exactly out to at least 4 decimal places (~ 0.1 km) for all the grid points. Both algorithms assume spherical earth transformation equations. Therefore the same conclusions as in Step 3 can be made regarding the spatial displacement errors between the NEXRAD HRAP grid and the GENHRAP HRAP grid.

V. Conclusions

The WSR-88D PPS software that remaps polar rainfall estimates to the HRAP coordinate system was examined in this study. It was found that the algorithm averages rainfall estimates for polar grid cells whose centers fall within an HRAP grid box. An HRAP grid box is defined as the area enclosed by four contiguous HRAP grid points (I, J) where I and J are integers. Thus the rainfall estimates are actually centered over the grid locations (I+0.5, J+0.5) that fall midway between the integer-multiple grid points and *not* centered over the points (I, J) as defined in the written NEXRAD AEL specifications document.

A comparison was performed to determine the spatial registration compatibility of the HRAP grid as defined within the NEXRAD software with the HRAP grid as defined in the NWS's Stage III Precipitation Processing and River Forecast System software. Three radars were chosen for analysis: San Diego, CA, Denver, CO, and Caribou, ME. It was found that the two grids align properly in terms of their indexing (i.e., a particular NEXRAD HRAP grid coordinate index is properly registered relative to that same grid point index within the Stage III SBLLGD software), however there are small grid point displacement errors averaging about 0.3 km over the 230 km scanning domain of the WSR-88D between the two algorithms. The errors are small near the radar (~0.02 km) and increase approximately linearly with range to about 0.3-0.5 km at 230 km range depending on the radar. The mean displacement error increases for radars located farther to the north, and it is about 0.2-0.3 km which is less than or approximately equal to 1/10th of an HRAP grid cell size. This displacement is probably insignificant for most applications, especially considering the simplistic (though efficient) approach that the PPS uses to map polar grid cells into the HRAP grid using the cell centers.

VI. References

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Appendices

Appendix A. Selected pages from the NWS River Forecast System User's Manual describing the HRAP grid.

VI.3.3C DESCRIPTION OF ALGORITHMS USED BY PPINIT

This section describes the algorithms used in PPINIT to compute parametric values. The algorithms described are those used to:

- o define basin boundary parameters
- o compute weights for precipitation, temperature and PE stations
- o determine appropriate MDR boxes for stations or basins

NWSRES/HRAP Grid Coordinate System

Several of the algorithms used are derived from simple geometric principles. The most basic element common to all the geometric computations is the grid coordinate system used to identify the locations of stations and basin boundaries. The grid used is the same coordinate system used by the Hydrologic Rainfall Analysis Project (HRAP) and is the grid system used for precipitation estimations from the WSR-88D radars. The grid is based on a polar stereographic map projection with a standard latitude of 60° North and standard longitude of 105° West. The mesh length at 60° North latitude is 4.7625 km. Mesh lengths for other latitudes can be computed from:

$$zmesh = 4.7625 / ((1 + \sin(60)) / (1 + \sin(xlat))) \quad (1)$$

where zmesh is the mesh length in KM
xlat is the latitude

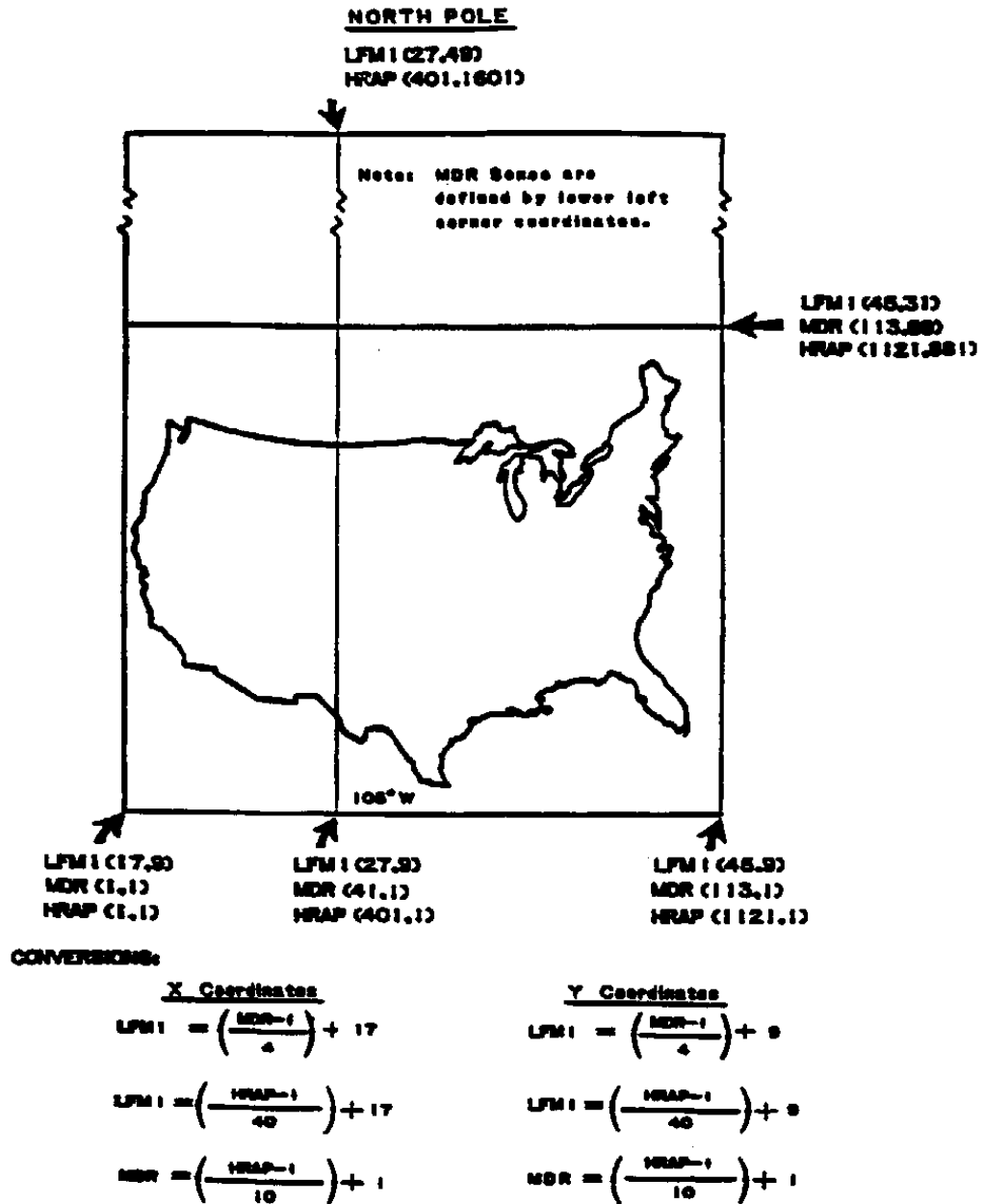
The grid is positioned such that coordinates (401,1601) are at the North Pole, resulting in all positive coordinates within the United States. The coordinates of a point P(X,Y) are computed as follows:

$$\begin{aligned} RE &= (EARTH * (1 + \sin(60))) / xmesh \\ R &= (RE * \cos(xlat)) / (1 + \sin(xlat)) \\ WLONG &= xlon + 75 \\ X &= R * \sin(WLONG) + 401 \\ Y &= R * \cos(WLONG) + 1601 \end{aligned}$$

where EARTH is the radius of the earth (6371.2 KM)
xmesh is the mesh length at 60° latitude (4.7625 KM)
xlat is the latitude of point to be converted (decimal degrees)
xlon is the longitude of point to be converted (decimal degrees)

The orientation and mesh length of the grid was selected such that it contains the National Meteorological Center (NMC) Limited Fine Mesh I (LFM I) and the NWS Manually Digitized Radar (MDR) grids as subsets. The NWSRFS/HRAP grid mesh length is 1/40 the size of the LFM I mesh length and 1/10 the size of the MDR mesh length. The coordinate systems are shown in Figure 1.

Figure 1. LFM I, MDR and HRAP Coordinate Systems



Appendix B. Selected pages from the NEXRAD Algorithm Report describing the specifications for mapping of polar rainfall estimates to the HRAP grid.

1.0 PROLOGUE

1.1 FUNCTIONAL DESCRIPTION

The PRECIPITATION PRODUCTS [021] creates the Hydrometeorological products from hourly and scan-to-scan accumulations generated by the PRECIPITATION ACCUMULATION [019] algorithm and adjusted by the current BIAS computed by the PRECIPITATION ADJUSTMENT [020] algorithm if FLAG (apply BIAS) is set. The products generated are: (1) an hourly running total or clock hour accumulation on both a 1/40th Limited Fine Mesh (LFM) rectangular (approximately 4 km by 4 km) grid and a 2 km by 2 km resolution grid; (2) a three hour total accumulation; and (3) a storm total accumulation. The latter two products are on a 2 km by 2 km resolution grid. The 2 km by 2 km resolution products are intended primarily for graphical display while the 1/40th LFM grid product is an array product intended primarily for transmission and numerical use at external computer facilities.

The hourly running totals or clock hour totals on the 1/40th LFM grid are obtained by determining the mean of all adjusted ACCUMULATION SCAN (Hourly) sample volumes whose polar coordinate centers fall within each 1/40th LFM grid box. At the far ranges where no sample volume centers fall inside a box, the sample volume value at the sample volume whose center is closest to the center of the grid box becomes the value at the grid box. Annotations are automatically added to identify the product and to provide information related to how the data used to generate this product were processed.

The hourly running totals or clock hour totals on the 2 km by 2 km grid are determined using the same approach used for 1/40th LFM grid boxes above. The data are then scaled to 16 levels for use as a display and annotations are added automatically to produce the 2 km by 2 km PRODUCT (Hourly Precipitation).

The three clock hour totals on the 2 km by 2 km grid are computed hourly by summing the available individual clock hour totals for the past three hours. At least two of the three hours of data must be available and missing periods should be noted. The data are then scaled to 16 accumulation levels for use as a display, and annotations are added automatically to produce the 2 km by 2 km PRODUCT (Three Hour Precipitation).

The storm total (total precipitation since the last one hour break in significant precipitation) on the 2 km by 2 km grid is generated whenever certain scan-to-scan accumulation parameters are exceeded. It is then updated using each ACCUMULATION SCAN (Scan-to-Scan) received until being reset after a one hour break in significant precipitation. If FLAG (apply BIAS) is set, the ACCUMULATION SCAN (Scan-to-Scan) are adjusted using the computed BIAS. The data is then scaled to 16 levels for use as a display and annotations are added automatically to produce the new 2 km by 2 km PRODUCT (Storm Total Precipitation).

1.2 SOURCE

The PRECIPITATION PRODUCTS algorithm was developed by the Radar Hydrology Group of the National Weather Service's Hydrologic Research Laboratory. This algorithm is based on experiences gained through the use of real-time rainfall estimation from the D/RADEX system, the GATE project, and other experimental projects as well as an in-depth analysis of ways with which weather radar data could be better used for hydrometeorological purposes.

Ahnert, P.R., M.D. Hudlow, and E.R. Johnson, 1984: Validation of the "on-site" Precipitation Processing System for NEXRAD. Preprints, 22nd Radar Meteor. Conf., AMS, Boston, Mass.

Ahnert, P.R., M.D. Hudlow, E.R. Johnson, D.R. Greene, and M.R. Dias, 1983: Proposed "on-site" precipitation processing system for NEXRAD. Preprints, 21st Radar Meteor. Conf., AMS, Boston, Mass.

Hudlow, M.D., D.R. Greene, P.R. Ahnert, W.F. Krajewski, T.R. Sivaramakrishnan, M.R. Dias, and E.R. Johnson, 1983: Proposed off-site precipitation processing system for NEXRAD. Preprints 21st Radar Meteor. Conf., AMS, Boston, Mass.

1.3 PROCESSING ENVIRONMENT

The ARRAY PRODUCT (Digital Precipitation) on the 1/40th LFM grid provides hourly running total or clock hour total precipitation accumulation estimates in a digital array format to support hydrometeorological requirements for numerical use of precipitation data in computers external to the NEXRAD RPG. This format is not compatible with PUP displays. In addition to the precipitation array data, an extensive set of annotations (IDENTIFIER INFORMATION and SUPPLEMENTAL DATA) will be included automatically as part of this product. This information is intended for use in higher level (regional/national) processing to identify certain characteristics about the data up to that point in the processing stream. It will be used as part of the information for accomplishing more discriminating quality control functions at the higher level of processing. Because some users may not require the full set of annotations, the option must exist to request the product with abbreviated or complete annotations. Abbreviated annotations should include only the product identification information and a list of missing periods. Also for certain applications it will be useful to obtain selected annotations (SUPPLEMENTAL DATA) including the RATE (1/4th LFM Grid Box) without the hourly precipitation totals themselves. This option must also be provided.

The Limited Fine Mesh (LFM) grid is a rectangular grid commonly used by the National Weather Service which is based on a polar stereographic projection. An LFM grid box represents an area whose size and shape varies with latitude. Therefore the size and shape of the grid boxes will vary slightly over the area covered by the radar and even more from radar to radar (3.5 to 4.5 km over the conterminous U.S. for the 1/40th LFM grid. The

1/40th LFM grid boxes used here are defined to have 1/40th LFM grid points as their centers and a mesh length of 4.7625 km at 60° N latitude. The information required to generate the grid for each site are the latitude and longitude of the radar, the mesh length at 60° N latitude (the standard latitude), and the standard longitude (105° W)).

In order to cover the radar umbrella out to 230 km even at the lower latitudes of the conterminous United States, a 131 by 131 array of 1/40th LFM grid boxes will be required. This array will always be 131 by 131 regardless of the latitude of the site. This grid should be positioned in such a way that the radar site falls within the grid box (66,66). The ARRAY PRODUCT (Digital Precipitation) must be compacted (e.g., elimination of all 0 rows, run length encoding of rows) to reduce storage and especially communications loadings. Compaction must be done in such a way that the source 131 by 131 array can be reconstructed with the use of nominal computer resources. The 1/4th LFM area averaged precipitation rate data (8 coded precipitation rate levels) for each scan used to generate the ARRAY PRODUCT (Digital Precipitation) will normally be automatically included as part of the annotations (SUPPLEMENTAL DATA) to the ARRAY PRODUCT (Digital Precipitation). The values for the 13 by 13 1/4th LFM grid were computed by the PRECIPITATION RATE [018] algorithm. These must be compacted subject to the constraints specified above.

The 2 km by 2 km PRODUCT (Three Hour Precipitation) uses the 2 km by 2 km PRECIPITATION TOTALS (Hourly) for the last three clock hours. In order to provide these products on a consistent basis, the method used to save the 2 km by 2 km PRECIPITATION TOTALS (Hourly) must be safe, even from temporary system shutdowns and restarts.

The 2 km by 2 km PRODUCT (Storm Total Precipitation) uses the previous set of 2 km by 2 km PRECIPITATION TOTAL (Storm). Again, the method used to save these data must be safe, even from temporary shutdowns and restarts.

Appendix C. Selected pages from the NEXRAD B-5 Specifications document describing how polar rainfall estimates are mapped to the HRAP grid.

30.6 Polar/LFM Conversion

30.6.1 Description

An algorithm is provided to identify the subset LFM grid box (I,J coordinates) corresponding to each display plane polar radar coordinate. Grid boxes for the 1/4, 1/16, and 1/40 LFM grids centered on the radar site are identified for each polar radar coordinate. The methodology used is to first translate each polar radar coordinate to a corresponding latitude, longitude in a manner consistent with the projection of background map data to the NEXRAD display plane. A polar stereographic projection of latitude, longitude to the LFM grid plane (tangent to the earth at the North Pole) is then performed to identify the grid box. The equations given apply for a northern hemispherical grid. However, these equations are useable to about 30 degrees latitude into the southern hemisphere. The grid is oriented such that the J-axis is parallel with the specified prime meridian (105 degrees W) and has a 1/4 LFM

box size of 47.625 km true at 60 degrees latitude. For computational convenience, the I-axis for the grid is defined as positive to the right and the J-axis is positive down with the I, J coordinates of the North Pole set to 433,433. An earth's radius of 6371.221 km is used for the polar stereographic projection of latitude, longitude to the LFM grid plane.

It should be noted that the algorithm provided assumes a polar radar coordinate resolution of 1-km range by 1-degree azimuth with the azimuths centered on 0.5 degree, 1.5 degrees, etc. and range extending to 460 km. A range resolution of 2 km may be used with coarse resolution products where desirable to save throughput or simplify program design. Centering of the azimuths is arbitrary since the radar has no fixed azimuth position for any given radar beam. Grid boxes need not be defined beyond a range of 230 km for the 1/4 and 1/40 grids since the associated products are limited to this range.

30.6.2 Definition of Terms

L_S, λ_S	=	site latitude, longitude (deg)
L, λ	=	latitude, longitude of a given polar radar coordinate
R, B	=	range, bearing of a given polar radar coordinate
GI, GJ	=	global grid coordinates of a given polar radar coordinate in units of 1/4 LFM boxes (I positive to right, J positive down)
$I(M), J(M)$	=	local grid box number of a given polar radar coordinate for 1/4, 1/16, 1/40 LFM grids (M = 1,2,3, respectively). Upper left corner box of grid is numbered I,J = 1,1
I_P, J_P	=	global grid coordinates of North Pole in units of 1/4 LFM grid boxes ($I_P, J_P = 433, 433$)
GI_S, GJ_S	=	global grid coordinates of radar site in units of 1/4 LFM boxes
$I_S(M), J_S(M)$	=	global grid box number for box 0.0 of 1/4, 1/16, 1/40 local LFM grids (M = 1,2,3, respectively)
S	=	angular great circle distance between polar radar coordinate and radar site
K_C	=	grid scale factor = $\frac{2 * R * (1 + \sin 60^\circ)}{47.625} = 249.6348607$
R_S	=	earth's radius (= 6,371.221 km)

30.6.3 Procedure

1.0 Compute $\sin L_S, \cos L_S, \cos(\lambda_S + 105^\circ), \sin(\lambda_S + 105^\circ)$

2.0 Compute Reference Grid Box Coordinates

$$GI_S = K_C \frac{\cos L_S}{1 + \sin L_S} \sin(\lambda_S + 105^\circ) + I_P$$

$$GJ_S = K_C \frac{\cos L_S}{1 + \sin L_S} \cos(\lambda_S + 105^\circ) + J_P$$

2.1 Compute Grid Box Number for box, 0.0 of local grids

$$I_S(1) = \text{INT}(GI_S) - 7$$

$$J_S(1) = \text{INT}(GJ_S) - 7$$

$$I_S(2) = \text{INT}(GI_S) - 49$$

$$J_S(2) = \text{INT}(GJ_S) - 49$$

$$I_S(3) = \text{INT}(GI_S) - 66$$

$$J_S(3) = \text{INT}(GJ_S) - 66$$

where $I_S, J_S(1,2,3)$ are box numbers for 1/4, 1/16 and 1/40 LFM grids

3.0 Initialize Bearing

$$B = -0.5 \text{ degree}$$

4.0 Do for each bearing (360 times)

$$B = B + 1 \text{ degree}$$

$$R = -1/2 \text{ km}$$

compute $\sin B, \cos B$

4.1 Do for each range (460 times)

$$R = R + 1 \text{ km}$$

$$\sin S = \frac{R}{6.380} \left(1 - \frac{135R}{6.380^2} \right)$$

$$\cos \Delta\lambda = (1 - \sin^2 \Delta\lambda)^{1/2}$$

where: $\Delta\lambda = \lambda - \lambda_s$

$$\cos S = (1 - \sin^2 L)^{1/2}$$

$$\sin L = \sin L_S \cos S + \cos L_S \sin S \cos B$$

$$\cos L = (1 - \sin^2 L)^{1/2}$$

$$\sin \Delta\lambda = \sin S \sin B / \cos L$$

$$R_L = K_C * \frac{\cos L}{1 + \sin L}$$

$$GI = R_L \sin \Delta\lambda \cos (\lambda_s + 105^\circ) + R_L \cos \Delta\lambda \sin (\lambda_s + 105^\circ) + I_p$$

$$GJ = R_L \cos \Delta\lambda \cos (\lambda_s + 105^\circ) - R_L \sin \Delta\lambda \sin (\lambda_s + 105^\circ) + J_p$$

4.1.1 Do for M = 1,2,3

$$I(M) = \text{INT}[GI * K_A(M)] - I_S(M)$$

$$J(M) = \text{INT}[GJ * K_A(M)] - J_S(M)$$

where $K_A = 1, 4, 10$ or $M = 1, 2, 3$ respectively

4.2 End of range computation

5.0 End of bearing computation

Notes:

1. For 1/4 LFM, box MM has I,J = 7,7
For 1/16 LFM box MMA has I,J = 49,49
2. I,J values outside of the following ranges should be set to Not Applicable:
For 1/4 LFM, I,J = 1,1 to 13,13
For 1/16 LFM, I,J = 1,1 to 100,100
For 1/40 LFM, I,J = 1,1 to 131,131
3. To minimize storage requirements, local grid box numbers may be encoded into single numbers. To encode by row first, use $I + N_G(J-1)$. To encode by column first, use $J + N_G(I - 1)$
 $N_G = 25, 100, 131$ for 1/4, 1/16, 1/40 LFM grids respectively.

30.6.4 Additional Considerations

At longer ranges and at lower latitudes, the grid box size becomes smaller relative to the polar radar coordinate resolution. In such circumstances, it is possible that certain grid boxes will not be mapped to any polar radar coordinate at the completion of the above algorithm. For these cases, the closest coordinate will be used. Thus, for example, if box i,j on the local grid has no cell mapped, the range, azimuth of the closest coordinate is defined as:

$$R = \text{INT}(RC_{ij}) + 0.5$$

$$B = \text{INT}(\theta C_{ij}) + 0.5$$

where:

$$RC_{ij} = \text{Range to center of box } i,j \text{ (km)}$$

$$\theta C_{ij} = \text{Azimuth to center of box } i,j \text{ in degrees (0° to 359.999°)}$$

A method for computing RC_{ij} , θC_{ij} is given in Section 30.6.4.1.

30.6.4.1 Computation for Coordinates of Grid Box Center

Coordinates of box centers (relative to pole) in units of 1/4 LFM boxes can be computed as:

$$CI_i = AI(M) + B(M) * i$$

$$CJ_j = AJ(M) + B(M) * j$$

where M = 1,2,3 for 1/4, 1/16, 1/40 grids

$$i,j = 1,1 \text{ to } 13,13 \text{ for } 1/4 \text{ grid}$$

$$i,j = 1,1 \text{ to } 100,100 \text{ for } 1/16 \text{ grid}$$

$$i,j = 1,1 \text{ to } 131,131 \text{ for } 1/40 \text{ grid}$$

AI, AJ, B are constants defined as:

$$AI(1) = I_s(1) - I_p + 0.5$$

$$AJ(1) = J_s(1) - J_p + 0.5$$

$$AI(2) = (I_s(2) - 4I_p + 0.5) / 4$$

$$AJ(2) = (J_s(2) - 4J_p + 0.5) / 4$$

$$AI(3) = (I_s(3) - 10I_p + 0.5) / 10$$

$$AJ(3) = (J_s(3) - 10J_p + 0.5) / 10$$

$$B(1), (2), (3) = 1, 1/4, 1/10, \text{ respectively}$$

Latitude, longitude of box i,j center is then:

$$L_{i,j} = 90^\circ - 2 \tan^{-1} \left[\frac{(CI_i^2 + CJ_j^2)^{1/2}}{K_C} \right]$$

$$\lambda_{i,j} = -105^\circ + \tan^{-1} \left[\frac{CI_i}{CJ_j} \right] \quad (\text{use two-variable arctangent function})$$